# Liquidity, Monetary Policy and Unemployment*

Mei Dong† Sylvia Xiaolin Xiao‡

February 17, 2016

**Abstract**

We discover a consumption channel of monetary policy in a model with money and government bonds. When the central bank purchases government bonds (short-term or long-term), it drives up the price of bonds and lowers the return on bonds. The lower return on bonds has a direct negative impact on consumption by households that hold bonds, and an indirect negative impact on consumption by households that hold money. As a result, firms earn less profits from production, which leads to higher unemployment. The existence of such a consumption channel can help us understand the effects of unconventional monetary policy.

**JEL classification**: E24, E40, E50

**Key words**: interest rate, monetary policy, consumption, unemployment

*We would like to thank Chris Edmond, Pedro Gomis-Porqueras, Susumu Imai, Benoit Julien, Chao Gu, Tim Kam, Gordon Menzies, Guillaume Rocheteau, Jeff Sheen, Chi Chung Siu, John Wooders, Stephen Willamson, Randall Wright, Yajun Xiao and seminar participants at Deakin U., Monash U., RMIT, Tsinghua U., U. of Adelaide, U. of Melbourne, U. of New South Wales, U. of Queensland, U. of Technology, Sydney, U. of Wisconsin-Madison, FRB of St. Louis, the 26th Australian PhD Conference in Economics and Business, Sydney Macro Reading Group and 2014 Chicago Fed Summer Workshop on Money, Banking, Payments and Finance for helpful suggestions and comments. Dong acknowledges financial support under Australian Research Council's DECRA scheme (project number: DE120102589). Xiao acknowledges the hospitality and support during her visits at U. of Wisconsin-Madison, FRB of St. Louis and U. of Melbourne during Jan., 2014 to June, 2015, and the funding support from Dan Searle Fellowships since August, 2015.

†Department of Economics, University of Melbourne; Email: mei.dong@unimelb.edu.au.

‡Department of Economics, University of Wisconsin-Madison; E-mail: xxiao37@wisc.edu.

# 1 Introduction

We develop a model with money and government bonds to study how central bank's purchases/sales of government bonds affect output and unemployment through a consumption channel. Conventional monetary policy generally targets some short-term interest rates through open market operations (OMOs). During the recent Great Recession, targeted short-term interest rates in several advanced economies have been cut close to zero.[1] This limits central banks' ability to further lower the short-term interest rate to stimulate the economy. Instead of targeting short-term interest rates, the central banks of the U.S., Japan and some European countries all conducted *unconventional monetary policy* by either purchasing long-term government bonds or other government-guaranteed private securities in financial markets. The goal is to directly lower long-term interest rates in financial markets to stimulate economic activity and job creation.

To understand the effects of this unconventional monetary policy, we build a general equilibrium model following recent developments in monetary theory. There are three key elements of our model. The first is that money and bonds are valued by households because they can facilitate transactions in the retail goods market. The coexistence of money and bonds makes the model suitable to consider OMOs as central bank's outright purchases or sales of government bonds. Monetary policy can affect households' portfolio decisions if the policy changes the relative return of these assets. The second key element is that the model generates unemployment. A labor

[1] In 2008, the Federal Reserve Bank of the U.S. cut the Federal Fund Rate to the zero lower bound. This zero-lower-bound problem is also observed in Japan and various European countries. In Japan, as early as in 1995, before the outbreak of 1997 Asian Financial Crisis, Bank of Japan cut the short-term interest rate almost to zero, which lasted until January, 2016, and then cut it to negative after that. After the Great Recession, the European Central Bank (ECB) cut the short-term target rate to the zero lower bound, and then to negative, firstly in June, 2014, and furthermore in Sept., 2014. Switzerland has followed a similar path, cutting the short-term target rate from the zero lower bound to negative, firstly in Dec. 2014, and furthermore in Jan., 2015. However, ECB decided to start its own Quantitative Easing in March, 2015, since the negative interest rate did not really stimulate credit to the economy, instead, may cause deflation.

market exists where vacant firms and unemployed households search. The third key element is that firms need to bring production in the labor market to the retail goods market for sale. Households can purchase goods for consumption in the retail goods market using money or government bonds. This link between labor market and goods market provides a channel through which monetary policy affects unemployment.

Our model builds on Berentsen et al. (2011). We first add short-term government bonds in addition to money so that households choose a portfolio of money and short-term bonds.[2] In such an environment, the central bank can adjust the relative supply of short-term government bonds as a monetary policy instrument, i.e., OMOs. We find that there exist different types of monetary equilibrium depending on the relative supply of short-term government bonds and the growth rate of money. OMOs can affect the economy only when the supply of bonds is relatively scarce, but not too scarce. In this case, the return on bonds is higher than that on money. Households that have access to bonds choose to use only bonds to trade in the retail market, and households that do not have access to bonds use money. When the central bank conducts a one-time purchase of government bonds and reduces the relative supply of bonds, the price of bonds increase and the short-term interest rate decreases. For households that can use bonds, the lower interest rate directly induces them to hold less bonds and consume less. As a result, firms' profits from selling to these households decrease. In the labor market, lower profits discourage firms from entering and raise unemployment. In the retail market, households face less trading opportunities and this will lower the marginal benefit of holding money. This general equilibrium effect indirectly makes households that use money hold less money and consume less, which will further lower firms' profits and raise unemployment.

[2] Here we can interprete "households" not only as buyers in the retail goods market, but as financial investors holding money, or a portfolio of money and bonds. As discussed later, implicitly, households also equally hold equity of firms, in our model. This also supports the interpretation of "financial investors". Anyhow, from the perspective of the real world, this broader definition of "households" makes more sense to show consumption channel of monetary policy in our paper.

We focus on the effects of OMOs through a consumption channel. That is, the change of bonds' return resulting from purchasing or selling bonds can affect the consumption decisions by households and this in turn can affect labor market outcome. A one-time purchase of government bonds has a negative impact on output and employment through the consumption channel. The conventional view is that central bank's purchase of government bonds would lower interest rates and thus stimulate investment. In our model, firms do not have any investment decisions to make and therefore this investment channel is absent. As this investment channel has been the identified in recent studies such as Rocheteau and Rodriguez-lopez (2014), we argue that the consumption channel is complementary to the investment channel.

In the basic model, it is possible that the economy is in a liquidity trap equilibrium where the short-term interest rate is zero. OMOs cannot further lower the short-term interest rate. During the Great Recession, several central banks choose to purchase long-term government bonds or other government-guaranteed private securities. To address the effects of this unconventional policy, we extend the basic model by adding long-term government bonds. Hence, when the short-term interest rate is zero, the central bank can buy or sell long-term government bonds to directly adjust the long-term interest rate. The extended model shows that it is natural for the central bank to resort to unconventional policy to affect the economy, when the short-term interest rate is zero. Through the consumption channel, unconventional policy lowers the long-term interest rate and households' consumption. As a result, equilibrium unemployment increases.

Our paper is related to two lines of research in the monetary search literature. The first line integrates monetary models with labor search models, as in Berentsen et al. (2011) and Bethune et al. (2015). In these papers, money serves as the medium of exchange and monetary policy is modeled as adjusting the growth rate of money supply. In our paper, we introduce government bonds into the model so that we can consider OMOs as an alternative monetary policy instrument. The second line of research involves explicit modeling of assets and liquidity. Williamson (2012) and Rocheteau et al. (2015) are most closely related to our paper. Also see Mahmoudi (2013) and Williamson (2013). Williamson (2012) develops a model with money, government bonds and equity to study the effects of both conventional and unconventional monetary policies. He models unconventional monetary policy as purchasing private equity. Rocheteau et al. (2015) focus on OMOs and show how market structures and liquidity properties of money and bonds matter for understanding the effects of OMOs. However, these papers do not consider unemployment.

Recently, Rocheteau and Rodriguez-lopez (2014) build a model with an over-the-counter financial market and a Mortensen-Pissarides labor market. They focus on the investment channel of monetary policy, by showing how monetary policy lowers the interest rate and stimulate investment demand. They find that if a shock triggers a lower acceptability of private equity as collateral, the central bank can mitigate this shock by purchasing private equity. Wen (2013) and Herrenbrueck (2013) develop models to understand unconventional monetary policy. They calibrate to U.S. data and find that unconventional policy can effectively stimulate investment under certain conditions. In their models, there is no explicit unemployment because production occurs in competitive markets.

The rest of the paper is organized as follows. Section 2 describes the model's environment. Section 3 introduces the basic model. Section 4 characterizes monetary equilibria and analyze the effects of OMOs. Section 5 focuses on unconventional monetary policy by extending the basic model to incorporate long-term government bonds. We conclude in Section 6.

## 2 Environment

Time is discrete and continues forever. As in Berentsen et al. (2011), there are three subperiods in each period: there is a labor market in the first subperiod; a goods market in the second subperiod, and a frictionless centralized market in the last subperiod. We refer to these three markets as MP, KW and AD markets hereafter.[3] There are two types of agents: firms and households, indexed by $f$ and $h$. The measure of households is 1, while the measure of firms is arbitrarily large, but not all firms are active. In addition, there exists a government which is a consolidated fiscal and monetary authority. All government asset transactions take place in the AD market. Figure 1 shows the timeline of a representative period.

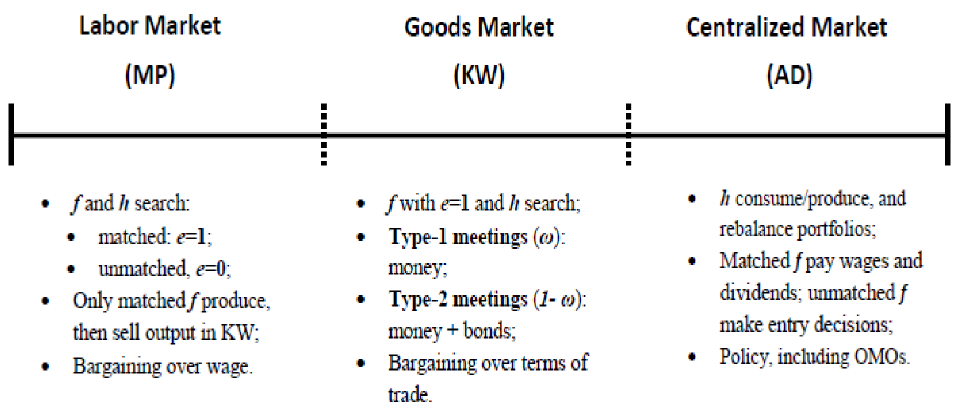


Figure 1: Timeline of a representative period

Let $e$ denote employment status: $e = 1$ if a household and a firm are matched, and $e = 0$ if unmatched. In the first subperiod, unemployed households and vacant firms match bilaterally to create a job. Once matched, the match produces output $y$. The wage $w$ is determined by generalized Nash bargaining. The match may break up at an exogenous rate $\delta$. If unmatched, the household is unemployed, and will receive unemployment benefits, $\kappa$. Households receive $w$ or $\kappa$ in the subsequent AD market.

[3] The labor market is based on Mortensen and Pissarides (1994); the goods market is based on Kiyotaki and Wright (1993); and the centralized market is an Arrow-Debreu competitive market.

In the second subperiod, all households enter the KW market as buyers of the KW goods. The utility from consuming $q$ units of the KW goods is $\upsilon(q)$, where $\upsilon(0) = 0$, $\upsilon'(0) = \infty$ and $\upsilon'' < 0 < \upsilon'$. Only firms with output from the MP market are active in the KW market as sellers while those unmatched firms skip the KW market. For active firms, the cost of producing $q$ units of the KW good is $c(q)$ in terms of $y$, where $c(0) = 0$, $c' > 0$ and $c'' \geq 0$. Buyers and sellers are matched randomly and bilaterally. The terms of trade are determined by bargaining in all meetings.

In the KW market, the roles of households and firms create the double coincidence problem. Since households cannot store any good, barter is impossible. Lack of commitment and lack of record-keeping imply that pure credit is not viable in the KW market. These frictions make assets necessary as a medium of exchange to facilitate trade. We assume that there are two permanent types of households, depending on whether households can use bonds in the KW market. A fraction $\omega$ of households can use only money, whom we label as type-1 households, i.e., $h = 1$. The rest fraction $1 - \omega$ of households can use both money and bonds.[4] We label these households as type-2 households, i.e., $h = 2$. We can view type-2 households as those who have access to financial assets.

All agents can enter the AD market in the last subperiod, where a good $x$ is produced and traded in this competitive market. We assume that this AD good is nonstorable. A household's utility from consuming $x$ units of AD goods is $x$. If $x$ is negative, it means that households produce $x$. This linear utility makes households' asset portfolios tractable. Firms with $e = 1$ sell inventory (if $c(q) < y$), rebalance asset portfolios and pay wages and dividends to households.[5] Firms with $e = 0$ can

[4] We model the liquidity difference between money and bonds through their roles as a medium of exchange. Money is more liquid than bonds as money can be used by all households, while only type-2 households can use bonds. Alternatively, one can model the liquidity difference between money and bonds through their roles as collateral (See Rocheteau et al., 2015). In that way, short-term bonds are less liquid as a collateral asset than money and long-term bonds are less liquid as a collateral asset than short-term bonds. We use the first interpretation in this paper for simplicity.

[5] As in Berentsen et al. (2011), we assume that firms are equally owned by households.

choose to create a new vacancy at a cost $k$. All agents discount between the AD market and the next MP market at rate $\beta$.

The government is active only in the AD market. In the baseline model, the government issues money and short-term government bonds. It can also buy or sell short-term bonds to conduct OMOs. Let $M_t$ be the money supply in period $t$ measured in the beginning of the period. The net growth rate of money supply is $\pi$. Short-term bonds are one-period nominal bonds. Bonds that are issued at some discount price in period $t$ would pay 1 unit of money in period $t+1$. Let $B_t$ be the supply of nominal bonds in period $t$. We focus on the steady states from now on, so we drop the time subscript when there is no confusion. It is useful to define the nominal interest rate of short-term bonds. From the Fisher equation, $i$ is defined as $1+i=(1+\pi)/\beta$.[6] Let $i_s$ be the nominal interest rates of short-term bonds,

$$1+i_s=\frac{\phi_m}{\phi_s},$$

where $\phi_s$ is the price of bonds in terms of the numeraire good $x$ and $\phi_m$ the price of money in terms of $x$. Later we use "+" to denote variables associated with the next period. As in Silveira and Wright (2010), we define the spread as the normalized nominal return difference between money and bonds. The spread of short-term bonds $s_s$ is

$$s_s=\frac{i-i_s}{1+i_s}. \tag{1}$$

# 3 Model

In this section, we analyze the value functions for households and firms, and then describe monetary policy. The value functions for the MP, KW and AD markets are

[6] Notice that when the central bank changes $\pi$, it is equivalent to changing $i$.

$U_e^j$, $V_e^j$, $W_e^j$, where $j \in \{1, 2, f\}$and $e \in \{0, 1\}$. We begin with the value functions for households and firms in the current AD market, and then move to the MP and KW markets next period.

## 3.1 Households

A household $h$ entering AD with type $j \in \{1, 2\}$, employment status $e$ and a portfolio of money and short-term bonds $(m, b_s)$, chooses $x$ and the portfolio $(\hat{m}, \hat{b}_s)$ for the next period,

$$W_e^j(m, b_s) = \max_{x, \hat{m}, \hat{b}_s} \left\{ x + (1-e)\chi + \beta U_e^j(\hat{m}, \hat{b}_s) \right\}$$

$$\text{st. } x + \phi_m \hat{m} + \phi_s \hat{b}_s + T = ew + (1-e)\kappa + \Delta + \phi_m m + \phi_m b_s,$$

where $\chi$ is the value of leisure. The LHS of the budget constraint is total expenditure, which includes the consumption of $x$, the value of money and bonds carried to next period, and taxes $T$. The RHS is total income, which includes wage $w$ or unemployment benefit $\kappa$, firms' dividend $\Delta$, and the value of money and bonds. Notice that the value of $b_s$ in terms of $x$ is $\phi_m b_s$ as 1 unit bond pays 1 unit money at maturity. The value of $\hat{b}_s$ in terms of $x$ is $\phi_s \hat{b}_s$ as new bonds are issued at the price $\phi_s$.

Substituting $x$ from the budget constraint into the value function, we obtain

$$(2) \qquad W_e^j(m, b_s) = I_e + \phi_m m + \phi_m b_s + \max_{\hat{m}, \hat{b}_s} \left\{ -\phi_m \hat{m} - \phi_s \hat{b}_s + \beta U_e^j(\hat{m}, \hat{b}_s) \right\},$$

where $I_e = ew + (1-e)(\kappa + \chi) + \Delta - T$. The envelop conditions give $\partial W_e^j(m, b_s)/\partial m = \partial W_e^j(m, b_s)/\partial b_s = \phi_m$. As in Lagos and Wright (2005), quasi-linear preferences in the AD market imply that $W_e^j$ is linear in $(m, b_s)$, and the choice of $(\hat{m}, \hat{b}_s)$ is independent of $(m, b_s)$.

For a household in the following MP market,

$$U_1^j(\hat{m},\hat{b}_s) = \delta V_0^j(\hat{m},\hat{b}_s) + (1-\delta)V_1^j(\hat{m},\hat{b}_s),$$
$$U_0^j(\hat{m},\hat{b}_s) = \lambda_h V_1^j(\hat{m},\hat{b}_s) + (1-\lambda_h)V_0^j(\hat{m},\hat{b}_s),$$

where $\lambda_h$ the endogenous job creation rate. Let $(u,v)$ denote the measure of unemployed households and vacancies. The matching function $\mathcal{N}(u,v)$ is constant return scale. We have $\lambda_h = \mathcal{N}(u,v)/u = \mathcal{N}(1,\tau)$, where $\tau = v/u$ measures the labor market tightness.

In the subsequent KW market, households become buyers while firms with $e=1$ become sellers. Each household is matched randomly with a firm. Given that the measure of households is 1 and the measure of firms with $e=1$ is $1-u$, the matching function is $\mathcal{M}(1,1-u)$, which is also assumed to be constant return to scale. Recall that there are two types of households. Type-1 households can use only money to trade. Their value function is

$$V_e^1(\hat{m},\hat{b}_s) = \alpha_h\left[\upsilon(q^1) + W_e^1(\hat{m}-d^1,\hat{b}_s)\right] + (1-\alpha_h)W_e^1(\hat{m},\hat{b}_s),$$

where $\alpha_h = \mathcal{M}(1,1-u)$ is the household's probability of meeting a firm and $(q^1,d^1)$ are the terms of trade in a meeting with a type-1 household. That is, the household uses $d^1$ units of money to exchange for $q^1$ units of KW goods. For type-2 households that can use both money and bonds, they have

$$V_e^2(\hat{m},\hat{b}_s) = \alpha_h\left[\upsilon(q^2) + W_e^2(\hat{m}-d^2,\hat{b}_s-\mu_s)\right] + (1-\alpha_h)W_e^2(\hat{m},\hat{b}_s),$$

where $(q^2,d^2,\mu_s)$ are the terms of trade in a meeting with a type-2 household. The household uses $d^2$ units of money and $\mu_s$ units of short-term bonds to exchange for $q^2$ units of KW goods.

Let $S^1 = \upsilon(q^1) - \phi_{m+} d^1$ and $S^2 = \upsilon(q^2) - \phi_{m+}(d^2 + \mu_s)$ be the trading surplus for type-1 and type-2 households, respectively. Using the linearity of $W_e^j$, we can rewrite $U_e^j$ for $j \in \{1, 2\}$ as

$$U_e^j(\hat{m}, \hat{b}_s) = \alpha_h S^j + \phi_{m+}(\hat{m} + \hat{b}_s) + \mathbb{E}W_{\hat{e}}^j(0, 0), \tag{3}$$

where $\mathbb{E}W_{\hat{e}}^j(0, 0)$ is the expectation with respect to next period's employment status. It is clear that $\partial U_e^j / \partial \hat{m}$ and $\partial U_e^j / \partial \hat{b}_s$ do not depend on the employment status. We then substitute (3) into the maximization problem of (2),

$$\begin{aligned} W_e^j(m, b_s) = {} & I_e + \phi_m (m + b_s) + \beta \mathbb{E} W_{\hat{e}}^j(0, 0) \\ & + \max_{\hat{m}, \hat{b}_s} \left\{ -\phi_m \hat{m} - \phi_s \hat{b}_s + \beta \left[ \alpha_h S^j + \phi_{m+}(\hat{m} + \hat{b}_s) \right] \right\}. \end{aligned} \tag{4}$$

From (4), it confirms that the choice of $(\hat{m}, \hat{b}_s)$ is independent of $e$ and $(m, b_s)$. Hence, households of the same type take the same portfolio of money and bonds out of each AD market.

## 3.2 Firms

In the AD market, the portfolio choices by firms are trivial. Firms would not carry any money or bonds out of the AD market since they would not use money or bonds in the subsequent MP or KW markets. For a matched firm with inventory $\xi$, money balances $m$ and short-term bonds $b_s$, its value function in the AD market is

$$W_1^f(\xi, m, b_s) = \xi + \phi_m m + \phi_m b_s - w + \beta U_1^f.$$

As firms do not carry any money or bonds into the MP and KW markets, we omit the state variables in $U_e^f$ and $V_e^f$ without loss of generality. Depending on a firm's employment status, the firm's value function in the following MP market is

$$U_1^f = \delta V_0^f + (1-\delta) V_1^f,$$
$$U_0^f = \lambda_f V_1^f + (1-\lambda_f) V_0^f,$$

where $\lambda_f = \mathcal{N}(u,v)/v = \mathcal{N}(1,\tau)/\tau$, is the endogenous job filling rate. As mentioned before, only firms with $e=1$ produce $y$ and participate in the subsequent KW, while unmatched firms skip the KW market.

In the KW market, a firm may meet a type-1 household or a type-2 household. The firm's value function is

$$V_1^f = \omega V_1^{f1} + (1-\omega) V_1^{f2}, \tag{5}$$

where

$$V_1^{f1} = \alpha_f W_1^{f1}[y - c(q^1), \phi_{m+} d^1, 0] + (1-\alpha_f) W_1^{f1}(y,0,0),$$
$$V_1^{f2} = \alpha_f W_1^{f2}[y - c(q^2), \phi_{m+} d^2, \phi_{m+}\mu_s] + (1-\alpha_f) W_1^{f2}(y,0,0).$$

Here $\alpha_f = \mathcal{M}(1, 1-u)/(1-u)$ is the firm's probability of trade. It costs a firm $c(q^j)$ units of goods produced in MP to produce $q^j$ units of KW goods. The firm can carry the rest $y - c(q^j)$ as inventory to the subsequent AD market. Using the linearity of $W_1^f$ in $(x, m, b_s)$, we rewrite (5) as

$$V_1^f = y - w + \alpha_f S_f + \beta U_1^f,$$

where $S_f = \omega[\phi_{m+} d^1 - c(q^1)] + (1-\omega)[\phi_{m+} d^2 + \phi_{m+}\mu_s - c(q^2)]$ is the firm's expected surplus from trading in the KW market.

The free entry condition in the AD market implies that firms with $e=0$ can choose to enter the AD market freely by paying the entry cost $k$. Thus we have

$$W_0^f = \max\left\{0, -k + \beta\lambda_f V_1^f + \beta(1-\lambda_f)V_0^f\right\},$$

where $V_0^f = W_0^f = 0$ in equilibrium. It follows that $k = \beta\lambda_f V_1^f$. As in Mortensen and Pissarides (1994), we can derive

$$k = \frac{\beta\lambda_f(y - w + \alpha_f S_f)}{1 - \beta(1-\delta)}. \tag{6}$$

Recall that firms pay out profits as dividends in the AD market. The aggregate profit by all firms is $(1-u)(y - w + \alpha_f S_f) - vk$. For a household that owns shares of all firms, the dividend income is $\Delta = (1-u)(y - w + \alpha_f S_f) - vk$.

## 3.3 Government

The government is a consolidated fiscal and monetary authority. We focus on monetary policy and treat fiscal policy as passive. Suppose that the government has a balanced budget in every period. The government budget constraint is

$$\phi_m (M - M_-) + \phi_s B_s + T = \phi_m B_{s-} + u\kappa. \tag{7}$$

Here "$-$" denotes variables associated with the previous period. The LHS of (7) shows the government's total revenue, which includes the value of newly issued money and bonds plus the tax revenue. The RHS represents total expenditure, which includes the value of previously issued government bonds and the unemployment benefit.

The central bank can either adjust the growth rate of money supply or the relative supply of money and bonds. We use $\sigma_s$ to denote the ratio of short-term government bonds to money. The central bank commits to monetary policy where money supply grows at $1+\pi$, and the ratio of short-term bonds to money is $\sigma_s$. That is,

$$\frac{M}{M_{-}} = 1+\pi \text{ and } \frac{B_s}{M} = \sigma_s. \tag{8}$$

By the Fisher equation, changing $\pi$ is equivalent of changing $i$. When the central bank adjusts $\sigma_s$, it resembles the type of OMOs where the central bank purchases or sells government bonds permanently. For example, decreasing $\sigma_s$ means that the central bank decreases the relative supply of bonds to the public. Williamson (2012) and Rocheteau et al. (2015) define OMOs in a similar way. To summarize, the central bank has two monetary policy tools: $i$ and $\sigma_s$.

# 4 Equilibrium

The terms of trade in three markets are determined as follows: agents are price takers in the AD market, and bargain over the terms of trade in the MP and KW markets. In this section, we solve for equilibrium conditions in all markets and define stationary monetary equilibrium. Then we use the model to analyze the effects of monetary policy.

## 4.1 Goods Market Equilibrium

When a firm and a household meet in the KW market, the terms of trade are determined by bargaining in all meetings. A generic way to define the bargaining solution is that for $j \in \{1, 2\}$, a household pays $g(q^j)$ in real terms to purchase $q^j$ units of the KW good, where $g(\cdot)$ depends on the specific bargaining protocol. For example, the bargaining protocol could be Kalai bargaining (Kalai, 1977) or generalized Nash bargaining. Let $\theta$ denote the household's bargaining power. If we consider Kalai bargaining, a type $j \in \{1, 2\}$ household and a firm bargain to

$$\max_{q^j, d^j, \mu_s^j} \left[ \upsilon\left(q^j\right) - \phi_{m+}(d^j + \mu_s \cdot \mathbf{I}^j) \right]$$

$$\text{st. } \upsilon\left(q^j\right) - \phi_{m+}(d^j + \mu_s \cdot \mathbf{I}^j) = \theta\left[\upsilon\left(q^j\right) - c\left(q^j\right)\right]$$

$$d^j \leq \hat{m} \text{ and } \mu_s \leq \hat{b}_s.$$

The indicator $\mathbf{I}^j$ is such that $\mathbf{I}^1 = 0$ and $\mathbf{I}^2 = 1$. The solution to Kalai bargaining is the following. In case that $d^j = \hat{m}$ and $\mu_s = \hat{b}_s$, we have

$$g(q^j) = \phi_{m+}(\hat{m} + \hat{b}_s \cdot \mathbf{I}^j) = (1-\theta)\upsilon(q^j) + \theta c(q^j). \tag{9}$$

In case that either $d^j < \hat{m}$ or $\mu_s < \gamma_s \hat{b}_s$, we have (9) and

$$q^j = q^* \text{ where } q^* \text{ solves } \upsilon'(q) = c'(q).$$

For now, we use the general bargaining solution where the payment for exchanging $q^j$ units of the KW good is $g(q^j)$. Note that another implicit constraint associated with the bargaining problem is $c\left(q^j\right) \leq y$. It means that a firm's supply of $q^j$ is restricted by the output $y$ produced in the labor market. Following Berentsen et al. (2011), we assume that $y$ is big enough so that this constraint never binds.

As argued in Lagos and Wright (2005), the bargaining solution must be $d^1 = \hat{m}$ and $q^1 = g^{-1}(\phi_{m+}\hat{m})$ in type-1 meetings. Given the bargaining solution, we move back to the AD market and solve for $(\hat{m}, \hat{b}_s)$ in (4) for type-1 households. Since type-1 households cannot use bonds for trading, the FOC with respect to $\hat{m}$ yields

$$i = \alpha_h(u)\lambda(q^1). \tag{10}$$

For type-2 households, they can use both money and bonds. Notice that the return on bonds must be no lower than the return on money. When $i_s > 0$, type-2 households would choose $\hat{m} = 0$ and an interior solution for $\hat{b}_s$ solves

$$s_s = \alpha_h(u)\lambda(q^2). \tag{11}$$

When $i_s = 0$, we have $s_s = i = \alpha_h(u)\lambda(q^2)$, which we will discuss in more detail later. We use $\lambda(q^j) = \upsilon'(q^j)/g'(q^j) - 1$ to denote the liquidity premium in a meeting with a type-$j$ household. In (10), the LHS $i$ is the marginal cost of spending 1 more unit of money for type-1 households, while the RHS is the marginal benefit of spending 1 more unit of money. Similarly, in (11), the LHS $s_s$ is the marginal cost of spending 1 more unit of short-term bonds for type-2 households, while the RHS is the marginal benefit of spending 1 more unit of short-term bonds. For any $u$, (10) and (11) determine $(q^1, q^2)$. Labor market and goods market are linked: more unemployment reduces the number of firms entering into the KW market and hence reduces the trading probability for households, which will further affect equilibrium $(q^1, q^2)$.

## 4.2 Labor Market Equilibrium

In the MP market, wage is determined by generalized Nash bargaining. Let $\eta$ be the bargaining power of a firm. Following Mortensen and Pissarides (1994), we can solve for

$$w = \frac{\eta[1-\beta(1-\delta)](b+\chi) + (1-\eta)[1-\beta(1-\delta-\lambda_h)]\,(y+\alpha_f S_f)}{1-\beta(1-\delta)+(1-\eta)\beta\lambda_h}. \tag{12}$$

Substituting (12) into (6), the free entry condition becomes

$$k = \frac{\eta\lambda_f(u)\,[y-\kappa-\chi+\alpha_f(u)S_f]}{r+\delta+(1-\eta)\lambda_h(u)}. \tag{13}$$

The flow condition in the labor market implies that $(1-u)\delta = \mathcal{N}(u,v)$. This implicitly defines $v = v(u)$. The free entry condition determines $u$, given $(q^1, q^2)$. This establishes another link between the labor market and goods market. Compared with the free entry condition in Berentsen et al. (2011), the firm's expected trading surplus $S_f$ in (13) is the expected surplus from trading with two types of households.

## 4.3 Equilibrium Allocation

In any monetary equilibrium, $i_s$ is endogenously determined given $(i, \sigma_s)$. The no-arbitrage condition implies that $i_s$ must not be lower than the nominal return of money (i.e., 0). In addition, $i_s$ cannot exceed $i$. Therefore,

$$0 \leq i_s \leq i.$$

We now define general equilibrium as follows.

**Definition 1** *Given $(i, \sigma_s)$, a stationary monetary equilibrium is a list $(q^1, q^2, i_s, u)$ such that (i) given $u$,$(q^1, q^2)$ solves (4), and $i_s$ satisfies (1);(ii) given $(q^1, q^2, i_s)$, $u$ satisfies (13); and (iii) asset markets clear.*

**Proposition 1** *Stationary monetary equilibrium exists if $k < \eta(y - \kappa - \chi)/(r + \delta)$ and $\pi \geq \beta - 1$.*

The proof follows Berentsen et al. (2011). If $k$ is too high, entry would be too costly for firms. Both labor market and goods market will shut down. Therefore, monetary equilibrium does not exist. Notice that there always exists a non-monetary equilibrium for any $k$. When monetary equilibrium exists, it need not be unique. Depending on the values of $(i, \sigma_s)$, there can be four types of monetary equilibrium. We analyze each type of equilibrium in the following.

1. Friedman Rule Equilibrium

When $i = 0$, the equilibrium allocation is $q^1 = q^2 = q^*$ and $u$ is solved from (13). For bonds to be valued, $i_s = 0$. It is costless for households to hold either money or bonds. We label this type of equilibrium as the Friedman rule equilibrium. The first-best $q^*$ can be achieved. The Friedman rule equilibrium exists if and only if $i = 0$, i.e., $\pi = \beta - 1$.

2. Liquidity Trap Equilibrium

When $i > 0$ and $\sigma_s$ is small, the equilibrium return on bonds $i_s$ is 0 and $s_s = i$. In this case, the very scarce supply of bonds makes the price of bonds very high. Money and bonds earn the same nominal return 0. They become perfect substitutes for type-2 households. We label this equilibrium as the liquidity trap equilibrium. The liquidity trap equilibrium exists if and only if $i > 0$ and $\sigma_s \leq (1 - \omega)/\omega$. From (10) and (11), $q^1 = q^2 = q^\ell$ is solved from $i = \alpha_h(u)\lambda(q^\ell)$. The model reduces to Berentsen et al. (2011). It follows that increasing $i$ would lower $q^\ell$ and raise $u$. For the effect of $\sigma_s$, OMOs are irrelevant because changing the relative supply of bonds does not affect the return on bonds and hence the equilibrium allocation.

3. Plentiful Bonds Equilibrium

When $i > 0$ and $\sigma_s$ is very big, it is possible that the equilibrium value of $i_s$ reaches its upper limit $i$ and $s_s = 0$. It is costless for households to hold bonds. Therefore, type-1 households are willing to hold any amount of bonds and type-2 households hold bonds at least to buy $q^*$ in the KW market. In terms of allocation, $q^1 = q^{1p}$ is solved from (10) and $q^2 = q^*$. This type of equilibrium exists if and only if $i > 0$ and $\sigma_s \geq \sigma^*$, where

$$\sigma^* = \frac{(1 - \omega)\, g(q^*)}{\omega g(q^{1p})}. \tag{14}$$

That is, when $\sigma_s$ is big enough, the supply of bonds is abundant and the return on bonds is high. We label this type of equilibrium as plentiful bonds equilibrium.

For the effects of monetary policy, increasing $i$ would lower $q^{1p}$, but have no effect on $q^2$. Inflation lowers firms' profits and raises unemployment. Changing $\sigma_s$ does not have any effect on the equilibrium allocation, so OMOs are again irrelevant in plentiful bonds equilibrium.

4. Equilibrium with Scarce Bonds

When $i > 0$, a more interesting case is that the equilibrium value of $i_s$ lies between 0 and $i$. In this case, the return on bonds is higher than that of money so that type-2 households would prefer to hold only bonds. Meanwhile, the return on bonds is not so high and type-2 households hold a finite amount of bonds. The aggregate demand for bonds in real terms is $(1-\omega)\,\phi_{m+}\hat{b}_s = (1-\omega)\,g\left(q^2\right)$, which equals the supply of bonds in real terms $\phi_{m+}B_+$. Type-1 households hold only money. The aggregate demand for money in real terms is $\omega\phi_{m+}\hat{m} = \omega g\left(q^1\right)$, which equals the supply of money in real terms $\phi_{m+}M_+$. Recall that $B_+/M_+ = \sigma_s$. It implies that

$$\frac{(1-\omega)\,g\left(q^2\right)}{\omega g\left(q^1\right)} = \sigma_s. \tag{15}$$

The equilibrium allocation $(q^1, q^2, i_s, u)$ are solved from (10), (11), (13), and (15). This type of equilibrium exists if and only if $i > 0$ and $(1-\omega)/\omega < \sigma_s < \sigma^*$. Compared with plentiful bonds equilibrium, the supply of bonds is relatively scarce and the return on bonds is low. Therefore, we label this type of equilibrium as scarce bonds equilibrium.

We summarize how different types of equilibrium exist depending on the values of $(i, \sigma_s)$ in Figure 2. Notice that $q^{1p}$ decreases as $i$ increases, which implies that $\sigma^*$ is an increasing function of $i$. The Friedman rule equilibrium exists when $i = 0$. For $i > 0$, the economy is in the liquidity trap equilibrium. Bonds are so scarce which leads to a high price and 0 return. As $\sigma_s$ increases and the relative supply of bonds increases, the economy moves to the scarce bonds equilibrium, where the return on bonds is positive but still lower than $i$. As $\sigma_s$ further increases and bonds become abundant, the price of bonds is low and the return on bonds reaches its upper limit $i$. In this case, holding an additional unit of bonds does not cost households anything.

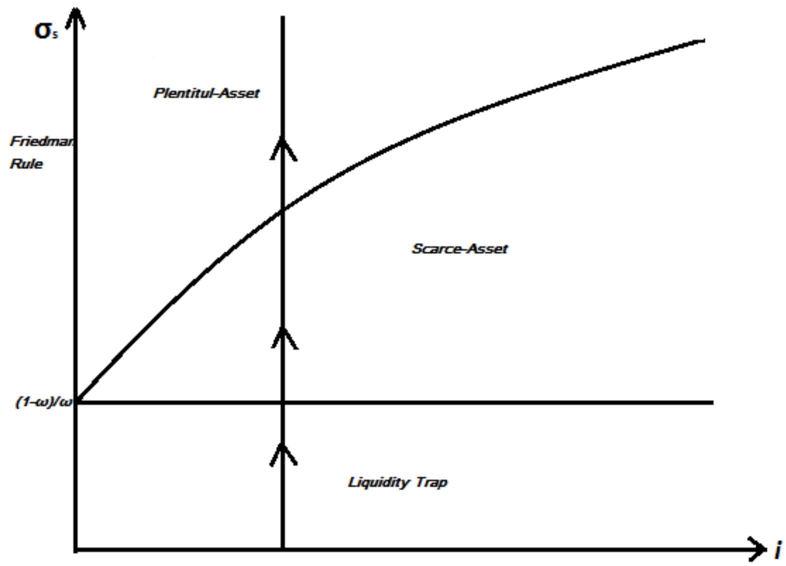


Figure 2: Four types of equilibrium depending on $(i, \sigma_s)$

It is more interesting to analyze the effects of $\sigma_s$. When $\sigma_s$ increases, the central bank increases the relative supply of bonds, which resembles a one-time sale of government bonds. A decrease in $\sigma_s$ resembles a one-time purchase of government bonds. From previous analysis, changing $\sigma_s$ affects equilibrium allocation only in the scarce bonds equilibrium.[7] In the following, we focus on the effects of monetary policy in the scarce bonds equilibrium.

**Proposition 2** *Consider the scarce bonds equilibrium. When monetary equilibrium is unique,* $\partial q^1/\partial \sigma_s > 0$, $\partial q^2/\partial \sigma_s > 0$, *and* $\partial u/\partial \sigma_s < 0$.

In general, there might be multiple equilibria due to the strategic complementarity between firm entry and household portfolio decisions. If there are multiple equilibria,

[7] It is true that changing $\sigma_s$ can move the economy from one type of equilibrium to another type of equilibrium. Among different type of equilibria, changing $\sigma_s$ affects allocation only in the scarce bonds equilibrium.

the comparative static results remain true in the equilibrium with the smallest $q^1$. Now take a decrease in $\sigma_s$ as an example. When $\sigma_s$ decreases, it means that the central bank decreases the relative supply of bonds. This open market purchase of bonds will drive up the price of bonds and lower the return on bonds. For type-2 households, a lower $i_s$ makes them hold less bonds and thus their consumption $q^2$ decreases. For firms, a lower $q^2$ tends to drive down their profits and reduce firms' incentives to create vacancies. In the labor market, unemployment would increase. For type-1 households, they are indirectly affected by this policy because more unemployment reduces the measure of sellers in the KW market. Type-1 households have less trading opportunities, so they respond by decreasing their holdings of real money balances in the KW market. It implies that $q^1$ should also decrease. A lower $q^1$would further reduce vacancies and raise unemployment.

In this model, we discover the effects of OMOs from the consumption channel. Whenever the central bank decreases the relative supply of bonds to lower the nominal interest rate, the lower interest rate is expected to stimulate investment. Instead of focusing on the effect of OMOs on investment, our model highlights how OMOs might affect households' consumption. We find that the lower interest rate would directly lower consumption by those who hold interest-bearing assets (type-2 households). Moreover, the lower interest rate would indirectly lower consumption by those who do not hold interest-bearing assets (type-1 households) through the rise of unemployment and the interaction between the labor market and the goods market. This would further increase unemployment. The open market purchase of bonds might stimulate investment, but it has a negative impact on consumption.

# 5 Unconventional Monetary Policy

In the section, we extend the model to include long-term government bonds to address the effects of unconventional monetary policy. These long-term bonds are perpetual bonds (like Consols) that pay 1 unit of money in every future period. The nominal interest rate on long-term government bonds $i_\ell$ and the spread $s_\ell$ are defined as

$$1 + i_\ell = \frac{1 + \phi_{\ell+}/\phi_{m+}}{\phi_\ell/\phi_m} \text{ and } s_\ell = \frac{i - i_\ell}{1 + i_\ell}, \tag{16}$$

where $\phi_\ell/\phi_m$ represents the nominal value of long-term bonds. In this new setting, type-1 households can still use only money whereas type-2 households can use money and both types of bonds in the KW market. When traded in the KW market, we assume that long-term bonds are not as liquid as short-term bonds so that type-2 households can use only a fraction $\gamma$ of their long-term bonds to buy KW goods.[8]

With long-term bonds, we update (4) as

$$\begin{aligned} W_e^j(m, b_s, b_\ell) &= I_e + \phi_m (m + b_s) + (\phi_m + \phi_\ell) b_\ell + \beta \mathbb{E} W_{\hat{e}}^j (0, 0) \\ &+ \max_{\hat{m}, \hat{b}_s, \hat{b}_\ell} \left\{ -\phi_m \hat{m} - \phi_s \hat{b}_s - \phi_\ell \hat{b}_\ell + \beta \left[ \alpha_h S^j + \phi_{m+}(\hat{m} + \hat{b}_s) + (\phi_{m+} + \phi_{\ell+}) \hat{b}_\ell \right] \right\}, \end{aligned} \tag{17}$$

where $S^1 = \upsilon(q^1) - \phi_{m+} d^1$ and $S^2 = \upsilon(q^2) - \phi_{m+}(d^2 + \mu_s) - (\phi_{m+} + \phi_{\ell+})\mu_\ell$, with $\mu_\ell$ denoting the amount of long-term bonds used by type-2 households in KW trades. For a firm, the expected trading surplus in the KW market is $S_f = \omega \left[\phi_{m+} d^1 - c(q^1)\right] + (1 - \omega)[\phi_{m+} (d^2 + \mu_s) + (\phi_{m+} + \phi_{\ell+})\mu_\ell - c(q^2)]$. The government budget constraint becomes

$$\phi_m (M - M_-) + \phi_s B_s + \phi_\ell (B_\ell - B_{\ell-}) + T = \phi_m B_{s-} + \phi_m B_{\ell-} + u\kappa. \tag{18}$$

[8] See Nosal and Rocheteau (2013) and Rocheteau et al. (2015) for similar approaches to model the liquidity difference of different assets.

In (18), newly issued long-term government bonds $\phi_\ell (B_\ell - B_{\ell-})$ contributes to government's revenue and payment incurred by the outstanding long-term bonds $\phi_m B_{\ell-}$ contributes to government's expenditure. With regard to monetary policy, the government commits to

$$\frac{B_\ell}{M} = \sigma_\ell, \tag{19}$$

in addition to (8). Therefore, monetary policy parameters now include $(i, \sigma_s, \sigma_\ell)$.

In the KW market, type-2 households can use money, short-term bonds and long-term bonds. Suppose that the terms of trade are determined by Kalai bargaining. A type-2 household and a firm bargain to

$$\max_{q^2, d^2, \mu_s, \mu_\ell} \left[ \upsilon\left(q^2\right) - \phi_{m+}(d^2 + \mu_s) - (\phi_{m+} + \phi_{\ell+})\,\mu_\ell \right]$$
$$\text{st. } \upsilon\left(q^2\right) - \phi_{m+}\left(d^2 + \mu_s\right) - (\phi_{m+} + \phi_{\ell+})\,\mu_\ell = \theta\left[\upsilon\left(q^2\right) - c\left(q^2\right)\right]$$
$$d^2 \leq \hat{m},\ \mu_s \leq \hat{b}_s \text{ and } \mu_\ell \leq \gamma \hat{b}_\ell.$$

Notice that $\mu_\ell \leq \gamma \hat{b}_\ell$ reflects that the household can use only a fraction $\gamma$ of long-term bonds in transactions. Type-2 households consume $q^*$ whenever any of the asset constraints is not binding. If all asset constraints are binding, $q^2$ is determined by $g\left(q^2\right) = \phi_{m+}(\hat{m} + \hat{b}_s) + (\phi_{m+} + \phi_{\ell+})\,\gamma \hat{b}_\ell$. In this case, there is an additional FOC with respect to $\hat{b}_\ell$ from (17) for type-2 households,

$$s_\ell = \gamma \alpha_h(u) \lambda(q^2). \tag{20}$$

It is immediate from (11) and (20) that

$$\frac{s_\ell}{s_s} = \gamma \text{ or } i_\ell = \frac{(1-\gamma)\,i + (\gamma + i)\,i_s}{1 + \gamma i + (1-\gamma)\,i_s}. \tag{21}$$

For both short-term bonds and long-term bonds to be held by type-2 households, the spread of long-term bonds must be lower than the spread of short-term bonds. That is, long-term bonds must have a higher return than short-term bonds. This type of positive term premium is also found in Williamson (2013) and Geromichalos et al. (2013).

Now depending on the values of $(i, \sigma_s, \sigma_\ell)$, there exist different types of monetary equilibrium. We leave the characterization of all types of equilibrium in the appendix and highlight two types of monetary equilibrium that are more relevant to the discussion of unconventional monetary policy.

When $i > 0$, $\sigma_s = 0$ and $(1-\omega)(1-\gamma) i / [\omega\gamma(1+i)] < \sigma_\ell < \sigma^* i / [\gamma(1+i)]$, the economy is an equilibrium where short-term bonds yield the same return as money but the return on long-term bonds dominates the return on short-term bonds. Notice that $\sigma^*$ is defined as in (14) and $q^{1p}$ in $\sigma^*$ is solved from (10). In this case, $0 = i_s < (1-\gamma) i / (1+\gamma i) < i_\ell < i$. It implies that short-term bonds are in a liquidity trap and long-term bonds are scarce. We label this equilibrium as a short-term bonds liquidity trap equilibrium because the nominal return on short-term bonds $i_s$ is 0. The central bank cannot further lower $i_s$ when conducting OMOs. Instead, the central bank can only rely on changing $i_\ell$ to affect the economy. We argue that this type of monetary policy resembles unconventional monetary policy conducted by central banks in U.S. and other advanced economies in the Great Recession.[9]

In this liquidity trap equilibrium, type-1 households hold money and type-2 households hold long-term government bonds to trade in the KW market. It follows that $g(q^2) = (\phi_{m+} + \phi_{\ell+}) \gamma \hat{b}_\ell$. We gather the equilibrium conditions (10), (13), (20), and the market clearing condition (19) that determine equilibrium $(q^1, q^2, u, s_\ell)$. Notice

[9] Our model focuses on the steady state long run equilibrium. One may question that unconventional monetary policy should be considered as short run stabilization policy. Given that the Fed has implemented this type of unconventional monetary policy for more than 7 years and the Bank of Japan has used similar policies for about two decades, we argue that it is useful to understand the long run implications of such policies.

that (19) implies that

$$g\left(q^2\right) = \frac{\omega\gamma\sigma_\ell\left(1+i_\ell\right)}{\left(1-\omega\right)i_\ell}g\left(q^1\right), \tag{22}$$

where $i_\ell$ is a function of $s_\ell$ from (16). We summarize the effects of changing $\sigma_\ell$ on $(q^1, q^2, u)$ in the following proposition.

**Proposition 3** *Consider the short-term bonds liquidity trap equilibrium. When monetary equilibrium is unique,* $\partial q^1/\partial\sigma_\ell > 0$, $\partial q^2/\partial\sigma_\ell > 0$, *and* $\partial u/\partial\sigma_\ell < 0$.

When the short-term interest rate $i_s$ is zero, it is natural for the central bank to rely on unconventional monetary policy by adjusting the relative supply of long-term government bonds to affect the long-term interest rate $i_\ell$. We find that lowering $\sigma_\ell$ can reduce $i_\ell$ as expected. The central bank's purchase of long-term government bonds reduces the supply of long-term bonds and raises the price of these bonds. Therefore, the long-term interest rate decreases. Through the consumption channel, a lower $i_\ell$ induces households to hold less bonds and consumption of $q^2$ decreases. Again through the general equilibrium effect, $q^1$ also decreases, which eventually leads to more unemployment. Qualitatively, the effects of lowering $\sigma_\ell$ are the same as the effects of lowering $\sigma_s$ in the basic model. However, since $g\left(q^2\right)$ takes a different form with long-term bonds, the exact magnitudes of these effects would not be the same as in the basic model.

Another equilibrium that is worth discussing is the scarce bonds equilibrium where $0 < i_s < i_\ell < i$ and both types of bonds are valued by households. This case happens when $i > 0$, $\sigma_s > 0$, $\sigma_\ell > 0$, $\sigma_s + \gamma\left(1+i\right)\sigma_\ell/\left[\left(1-\gamma\right)i\right] \geq \left(1-\omega\right)/\omega$ and $\sigma_s + \gamma\left(1+i\right)\sigma_\ell/i \leq \left(1-\omega\right)g\left(q^2\right)/\left[\omega g\left(q^1\right)\right]$. In the KW market, type-1 households still use money and type-2 households hold a portfolio of short-term and long-term government bonds. Therefore, $g\left(q^2\right) = \phi_{m+}\hat{b}_s + \left(\phi_{m+} + \phi_{\ell+}\right)\gamma\hat{b}_\ell$. The equilibrium conditions that characterize $(q^1, q^2, s_s, s_\ell, u)$ are (10), (13), (20), (21) and

$$g\left(q^2\right) = \left[\frac{\omega\sigma_s}{1-\omega} + \frac{\omega\sigma_\ell\gamma\left(1+i\right)}{\left(1-\omega\right)\left(i-s_\ell\right)}\right] g\left(q^1\right), \tag{23}$$

where (23) is derived from the asset market clearing conditions. In this equilibrium, both short-term and long-term bonds are held by type-2 households. Since long-term bonds are less liquid, their return is higher than the return on short-term bonds. The return premium stems from the liquidity difference between short-term and long-term bonds. Long-term bonds need to offer a higher return to compensate for being less liquid. If long-term bonds are as liquid as short-term bonds, i.e., $\gamma = 1$, we should have $i_s = i_\ell$ in the scarce bonds equilibrium.

Given that both $i_s$ and $i_\ell$ are positive, the central bank can adjust either $\sigma_s$ or $\sigma_\ell$ when conducting OMOs. In practice, it might be more common for central banks to change $\sigma_s$ when $i_s > 0$. However, historical evidence documented by D'Amico et al. (2012) reveals that the Fed did long-term bonds transactions between 1942 and 1951, which could directly affect the long-term interest rate. In our model, central banks have the freedom to choose between adjusting $\sigma_s$ or $\sigma_\ell$ only when the economy is in the scarce bonds equilibrium. We derive how $\sigma_s$ and $\sigma_\ell$ affect equilibrium allocation in Proposition 4.

**Proposition 4** *Consider the scarce bonds equilibrium. When monetary equilibrium is unique,* $\partial q^1/\partial\sigma_s > 0$, $\partial q^2/\partial\sigma_s > 0$, *and* $\partial u/\partial\sigma_s < 0$; *and* $\partial q^1/\partial\sigma_\ell > 0$, $\partial q^2/\partial\sigma_\ell > 0$, *and* $\partial u/\partial\sigma_\ell < 0$.

The qualitative effects of $\sigma_s$ and $\sigma_\ell$ on $(q^1, q^2, u)$ remain the same as before. Central bank's purchases of either short-term or long-term bonds increase demand for bonds and drive down the nominal interest rates. The lower interest rate induces type-2 households hold less bonds and cut back consumption of $q^2$. The decrease in $q^2$ has a negative impact of employment in the labor market, which indirectly reduces the trading opportunities of type-1 households in the KW market. Through this general equilibrium effect, type-1 households also hold less money and consume less $q^1$. As a result, employment further decreases.

The consumption channel indicates that central bank's purchases of government bonds always reduce consumption and increase unemployment. This may sound counter-intuitive, but the essence is that if monetary policy changes returns on assets and assets are not perfect substitutes, it could affect the portfolio choices by households and therefore affect the macroeconomy. Such a consumption channel exists only in models where households face non-trivial portfolio choices. Whether this channel is empirically important is still an open question.[10]

# 6 Conclusion

We build models where money and bonds coexist to examine the effects of monetary policy on macroeconomic performance such as consumption and unemployment. In the basic model with money and short-term government bonds, we find that central bank's purchases of bonds can lower the short-term interest rate. The lower interest rate induces households that use bonds to reduce their consumption. Households that do not use bonds also lower their consumption through a general equilibrium effect. The lower consumption by households reduces firms' profits and leads to higher unemployment in the economy. We highlight that the effects of such OMOs are through a consumption channel. When the economy's short-term interest rate is zero, it is natural for the central bank to resort to unconventional monetary policy to adjust the long-term interest rate. By purchasing long-term government bonds, the

[10] There is some empirical literature about the impacts of low interest rates during the Great Recession on household consumption and unemployment, such as Mian et al. (2013), Mian and Sufi (2014), and Keys et al. (2015). Particularly, Mian and Sufi (2014) show the housing net worth channel played a significant role in the sharp decline in the U.S. employment during 2007-2009. Although our model does not have the exact "housing net worth channel", the empirical results from Mian and Sufi (2014) still provide support for the link between household consumption and employment. See also Maggio et al. (2015).

long-term interest rate becomes lower, which again leads to lower consumption and higher unemployment.

When assessing the effectiveness of unconventional monetary policy, it is more common to focus on the investment channel where a lower interest rate could stimulate investment demand and output. Our model uncovers a new channel that works through consumption demand. In contrast to the effects on investment, the lower interest rate has negative effects of consumption and employment. We view this consumption channel as being complementary to the investment channel. It would be useful to construct models where both the consumption channel and the investment channel are present to evaluate the effectiveness of unconventional monetary policy. We leave this for future research.

# A Appendix A: Proof of Proposition 2

The equilibrium values of $(q^1, q^2, s_s, u)$ are determined by (10), (11), (15), and

$$H(u) = \omega\left[g\left(q^1\right) - c\left(q^1\right)\right] + (1-\omega)\left[g\left(q^2\right) - c\left(q^2\right)\right], \tag{24}$$

where (24) is derived from (13) and

$$H(u) = \frac{k\left[r + \delta + (1-\eta)\lambda_h(u)\right] - \eta\lambda_f(u)(y - \kappa - \chi)}{\eta\lambda_f(u)\alpha_f(u)}.$$

Since $\lambda_h'(u) < 0$, $\lambda_f'(u) > 0$ and $\alpha_f'(u) > 0$, we know that $H'(u) < 0$. We reduce the equations system to two equations (10) and (24) to solve for $(q^1, u)$, where $q^2$ is a function of $q^1$ through (15). Then $q^2$ is derived from (15) and $s_s$ can be derived from (11). Taking full derivation of (10) and (24), we have

$$\frac{\partial q^1}{\partial \sigma_s} = \frac{\omega\alpha_h'\lambda_1 g_1\left(g_2' - c_2'\right)}{D} \simeq -D,$$
$$\frac{\partial u}{\partial \sigma_s} = -\frac{\omega\alpha_h\lambda_1' g_1\left(g_2' - c_2'\right)}{D} \simeq D,$$

where

$$D = -\alpha_h\lambda_1' g_2' H' - \omega\alpha_h'\lambda_1\left[g_2'\left(g_1' - c_1'\right) + \sigma_s g_1'\left(g_2' - c_2'\right)\right].$$

If we graph (10) and (24) on the $(u, q^1)$ space, we know from (10)

$$\frac{dq^1}{du} = -\frac{\alpha_h'\lambda_1}{\alpha_h\lambda_1'} < 0.$$

It implies that (10) is downward sloping. Moreover, when $u \to 0$, $q^1$ is derived from $i = \alpha_h(1)\lambda\left(q^1\right)$, which should be a finite number. When $q \to 0$, $u$ should approach

1. From (24), we have

$$\frac{dq^1}{du} = \frac{g_2' H'}{\omega g_2' \left(g_1' - c_1'\right) + \omega \sigma_s g_1' \left(g_2' - c_2'\right)} < 0,$$

which means that (24) is also downward sloping. Moreover, when $u \to 0$, $H(u)$ approaches infinity. It follows that $q^1$ should approach infinity as well. The intersection of the two curves gives equilibrium $(u, q^1)$. If monetary equilibrium exists, there is at least one solution at which (24) is steeper than (10). If monetary equilibrium is unique or if we focus on the equilibrium with the smallest $q^1$, then it must be true that (24) is steeper than (10) at the equilibrium allocation. Mathematically,

$$-\frac{\alpha_h' \lambda_1}{\alpha_h \lambda_1'} > \frac{g_2' H'}{\omega g_2' \left(g_1' - c_1'\right) + \omega \sigma_s g_1' \left(g_2' - c_2'\right)}.$$

After rearranging, this exactly implies that $D < 0$.

We use (15) to derive

$$\frac{\partial q^2}{\partial \sigma_s} = -\frac{\omega g_1 \left[\alpha_h \lambda_1' H' + \omega \alpha_h' \lambda_1 \left(g_1' - c_1'\right)\right]}{(1-\omega) D}.$$

Notice that $D < 0$ implies that the $\alpha_h \lambda_1' H' + \omega \alpha_h' \lambda_1 \left(g_1' - c_1'\right) > 0$ and therefore we obtain

$$\frac{\partial q^1}{\partial \sigma_s} > 0, \ \frac{\partial q^2}{\partial \sigma_s} > 0, \text{ and } \frac{\partial u}{\partial \sigma_s} < 0.$$

From (11), we have

$$\frac{\partial s_s}{\partial \sigma_s} = -\frac{\omega \alpha_h g_1 \left\{\alpha_h \lambda_1' \lambda_2' H' + \alpha_h' \left[\omega \lambda_1 \lambda_2' \left(g_1' - c_1'\right) + (1-\omega) \lambda_1' \lambda_2 \left(g_2' - c_2'\right)\right]\right\}}{(1-\omega) D}.$$

The sign of $\partial s_s / \partial \sigma_s$ is not clear.

Recall that $s_s = (i - i_s) / (1 + i_s)$. It follows that $\partial s_s / \partial \sigma_s \simeq -\partial i_s / \partial \sigma_s$. Another way to see the sign of $\partial i_s / \partial \sigma_s$ is the following. From the equilibrium existence condition, we know that for any $i > 0$, liquidity trap equilibrium exists when $\sigma_s \in [0, (1-\omega)/\omega]$ and plentiful bonds equilibrium exists when $\sigma_s \in [\sigma_s^*(i), \infty)$. Also note that $i_s = 0$ when $\sigma_s = (1-\omega)/\omega$, and $i_s = i$ when $\sigma_s = \sigma_s^*$. When $\sigma_s \in [(1-\omega)/\omega, \sigma_s^*]$, the equilibrium is a scarce bonds equilibrium. From the equilibrium conditions, we have $i_s \to 0$ when $\sigma_s \to (1-\omega)/\omega$ and $i_s \to i$ when $\sigma_s \to \sigma_s^*$. When monetary equilibrium is unique, there is one $i_s$ for any given $\sigma_s$. If it is also true that there is one $\sigma_s$ for any given $i_s$, then we know that $i_s(\sigma_s)$ must be either strictly increasing or strictly decreasing.

To prove that there is one $\sigma_s$ for any given $i_s$ is equivalent to prove that the solution $(q^1, q^2, u)$ to (10), (26) and (24) is unique for any given $i_s$ or $s_s$. In general, it is not guaranteed that there exists a unique solution. However, when the solution is unique, we can argue that $i_s(\sigma_s)$ must be strictly monotonic. Given the values of the end points, it is only possible that $i_s(\sigma_s)$ is a strictly increasing function. That is

$$\frac{\partial i_s}{\partial \sigma_s} > 0 \text{ and } \frac{\partial s_s}{\partial \sigma_s} < 0.$$

# B Appendix B: Proof of Proposition 3

When $0 = i_s < i_\ell < i$, the return on short-term bonds is 0. The return on long-term bonds is positive. However, since long-term bonds are less liquid than short-term bonds, it is not clear how type-2 households choose among money, short-term bonds and long-term bonds. Recall that for both short-term bonds and long-term bonds to be held by type-2 households, we have (21). When $i_s = 0$, $i_\ell$ must be $(1-\gamma)i/(1+\gamma i)$ so that type-2 households hold both types of bonds. It follows that [1] when $0 = i_s < i_\ell < (1-\gamma)i/(1+\gamma i)$, type-2 households would not hold long-term bonds and monetary equilibrium is the same as the liquidity trap equilibrium in the model without long-term bonds; and [2] when $0 = i_s < (1-\gamma)i/(1+\gamma i) < i_\ell < i$, type-2 households hold only long-term bonds. In this case, short-term bonds and money are dominated by long-term bonds for type-2 households. If the economy is in this equilibrium, the government cannot further lower $i_s$ when conducting OMOs. Instead, the government can rely on changing $i_\ell$ to affect the economy. We argue that this type of monetary policy resembles unconventional monetary policy conducted by central banks in U.S. and other advanced economies.

To understand the effects of changing $\sigma_\ell$, we gather the equilibrium conditions (10), (24), (20), and

$$s_\ell = i - \frac{\omega\gamma\sigma_\ell\left(1+i\right)g\left(q^1\right)}{\left(1-\omega\right)g\left(q^2\right)}. \tag{25}$$

Here (25) is derived from (22). Substitute $s_\ell$ from (25) into (20),

$$i - \frac{\omega\gamma\sigma_\ell\left(1+i\right)g\left(q^1\right)}{\left(1-\omega\right)g\left(q^2\right)} = \gamma\alpha_h\left(u\right)\lambda\left(q^2\right). \tag{26}$$

Now we use (10), (26) and (24) to solve for $(q^1, q^2, u)$. Taking full derivation against these three equations, we have

$$\begin{aligned}
\frac{\partial q^1}{\partial\sigma_\ell} &= -\frac{\omega(1-\omega)\gamma(1+i)\alpha_h'\lambda_1 g_1(g_2'-c_2')}{D} \simeq D,\\
\frac{\partial q^2}{\partial\sigma_\ell} &= \frac{\omega(1+i)\gamma g_1[\alpha_h\lambda_1' H' + \omega\alpha_h'\lambda_1(g_1'-c_1')]}{D},\\
\frac{\partial u}{\partial\sigma_\ell} &= \frac{\omega(1-\omega)\gamma(1+i)\alpha_h\lambda_1' g_1(g_2'-c_2')}{D} \simeq -D,
\end{aligned}$$

where

$$\begin{aligned}
D = &\left(1-\omega\right)\gamma\alpha_h'\left(g_2'-c_2'\right)\left[\omega\sigma_\ell\left(1+i\right)\lambda_1 g_1' - \left(1-\omega\right)\alpha_h\lambda_1'\lambda_2 g_2\right]\\
&-\left(1-\omega\right)\left[\alpha_h\lambda_1' H' + \omega\alpha_h'\lambda_1\left(g_1'-c_1'\right)\right]\left[\gamma\alpha_h\left(\lambda_2' g_2 + \lambda_2 g_2'\right) - i g_2'\right].
\end{aligned}$$

To find the sign of $D$, we adopt the approach used in the proof of the basic model.

Instead of three equations, we reduce the system to two equations (10) and (24) to solve for $(q^1, u)$. From (10) and (20), we have $s_\ell = \gamma i \lambda_2 / \lambda_1$, which can be substituted into (25). In (24), we view $q^2$ as a function of $q^1$ solved from

$$g\left(q^2\right) = \frac{\omega\gamma\sigma_\ell\left(1+i\right)\lambda_1 g_1}{\left(1-\omega\right) i\left(\lambda_1 - \gamma\lambda_2\right)}. \tag{27}$$

From (10), we have

$$\frac{dq^1}{du} = -\frac{\alpha_h' \lambda_1}{\alpha_h \lambda_1'} < 0.$$

It means that in the $(u, q^1)$ space, (10) is downward sloping. Moreover, when $u \to 0$, $q^1$ is derived from $i = \alpha_h(1)\lambda(q^1)$, which should be a finite number. When $q \to 0$, $u$ should approach 1. From (24), we have

$$H'(u) = \omega\left(g_1' - c_1'\right)\frac{dq^1}{du} + \left(1-\omega\right)\left(g_2' - c_2'\right)\frac{dq^2}{du}, \tag{28}$$

where $dq^2/du$ can be derived from (27)

$$\frac{dq^2}{du} = \frac{\overbrace{\omega\gamma\sigma_\ell\left(1+i\right)\left(\lambda_1' g_1 + \lambda_1 g_1'\right) - \left(1-\omega\right) i\lambda_1' g_2}^{\Phi_1}}{\underbrace{\left(1-\omega\right) i\left(\lambda_1 g_2' - \gamma\lambda_2' g_2 - \gamma\lambda_2 g_2'\right)}_{\Phi_2}}\frac{dq^1}{du}. \tag{29}$$

We can show that $\Phi_1 > 0$ and $\Phi_2 > 0$. Substituting (29) into (28), we reach

$$\frac{dq^1}{du} = \frac{H'\Phi_2}{\omega\left(g_1' - c_1'\right)\Phi_2 + \left(1-\omega\right)\left(g_2' - c_2'\right)\Phi_1} < 0.$$

It implies that in the $(u, q^1)$ space, (24) is also downward sloping. Moreover, when $u \to 0$, $H(u)$ approaches infinity. It follows that $q^1$ should approach infinity as well.

We know that both (10) and (24) are downward sloping in the $(u, q)$ space. In addition, (24) must be above (10) at $u \to 0$. The intersection of the two curves gives equilibrium $(u, q^1)$. If monetary equilibrium exists and is unique (or we focus on the equilibrium with the smallest $q^1$), it must be the case that (24) is steeper than (10) at the equilibrium allocation. Mathematically, it must be true that

$$\frac{H'\Phi_2}{\omega\left(g_1'-c_1'\right)\Phi_2+\left(1-\omega\right)\left(g_2'-c_2'\right)\Phi_1}<-\frac{\alpha_h'\lambda_1}{\alpha_h\lambda_1'}. \tag{30}$$

After some algebra, one can find that (30) exactly implies that $D>0$. When $D>0$, one can also show that the numerator in $\partial q^2/\partial\sigma_\ell$ must be positive. We can conclude that

$$\frac{\partial q^1}{\partial\sigma_\ell}>0,\ \frac{\partial q^2}{\partial\sigma_\ell}>0,\ \text{and}\ \frac{\partial u}{\partial\sigma_\ell}<0.$$

# C Appendix C: Proof of Proposition 4

Suppose that $i_s>0$. When (21) is satisfied, we know that both short-term bonds and long-term bonds are held by type-2 households. It is easy to verify that $i_\ell<i$ iff $i_s<i$. We label this type of equilibrium as scarce bonds equilibrium. When $i_\ell<\left[\left(1-\gamma\right)i+\left(\gamma+i\right)i_s\right]/\left[1+\gamma i+\left(1-\gamma\right)i_s\right]$, long-term bonds are dominated by short-term bonds in terms of its return. Type-2 households hold only short-term bonds. This case is the same as the scarce bonds equilibrium in the basic model. When $i_\ell>\left[\left(1-\gamma\right)i+\left(\gamma+i\right)i_s\right]/\left[1+\gamma i+\left(1-\gamma\right)i_s\right]$, short-term bonds are dominated by long-term bonds in terms of its return. Type-2 households hold only long-term bonds. This case is the same as the liquidity trap equilibrium discussed above. It follows that when $i_s>0$, the only equilibrium where both short-term bonds and long-term bonds are valued features $0<i_s<i_\ell<i$.

In the scarce bonds equilibrium, $(q^1,q^2,s_s,s_\ell,u)$ are determined by (10), (11), (20), (24), and (23). We derive (23) using the asset market clearing conditions. Recall that $B_s/M=\sigma_s$ and $B_\ell/M=\sigma_\ell$. The asset market clearing conditions imply that

$$\frac{\left(1-\omega\right)\hat{b}_s}{\omega\hat{m}}=\sigma_s\ \text{and}\ \frac{\left(1-\omega\right)\hat{b}_\ell}{\omega\hat{m}}=\sigma_\ell.$$

Together with $g\left(q^1\right) = \phi_{m+}\hat{m}$ and $g\left(q^2\right) = \phi_{m+}\hat{b}_s + \left(\phi_{m+} + \phi_{\ell+}\right)\gamma\hat{b}_\ell$, we can reach (23). Notice that $s_s$ is determined by (11). One can find $s_\ell$ as a function of $\left(q^1, q^2, u\right)$ from (23) and substitute it into (20). Then we have three equations (10), (24) and

$$i - \frac{\omega\sigma_\ell\gamma\left(1+i\right)g\left(q^1\right)}{\left(1-\omega\right)g\left(q^2\right) - \omega\sigma_s g\left(q^1\right)} = \gamma\alpha_h\left(u\right)\lambda\left(q^2\right).$$

to solve for $\left(q^1, q^2, u\right)$.

Taking full derivation against these three equations, we have

$$\begin{aligned}
\frac{\partial q^1}{\partial\sigma_s} &= \frac{\omega(1-\omega)\alpha_h'\lambda_1 g_1(g_2'-c_2')\left(\gamma\alpha_h\lambda_2 - i\right)}{D} \simeq D,\\
\frac{\partial q^2}{\partial\sigma_s} &= -\frac{\omega g_1\left(\gamma\alpha_h\lambda_2 - i\right)\left[\alpha_h\lambda_1'H' + \omega\alpha_h'\lambda_1(g_1'-c_1')\right]}{D},\\
\frac{\partial u}{\partial\sigma_s} &= -\frac{\omega(1-\omega)\alpha_h\lambda_1' g_1(g_2'-c_2')\left(\gamma\alpha_h\lambda_2 - i\right)}{D} \simeq -D,\\
\frac{\partial q^1}{\partial\sigma_\ell} &= -\frac{\omega(1-\omega)\gamma(1+i)\alpha_h'\lambda_1 g_1(g_2'-c_2')}{D} \simeq D,\\
\frac{\partial q^2}{\partial\sigma_\ell} &= \frac{\omega(1+i)\gamma g_1[\alpha_h\lambda_1'H' + \omega\alpha_h'\lambda_1(g_1'-c_1')]}{D},\\
\frac{\partial u}{\partial\sigma_\ell} &= \frac{\omega(1-\omega)\gamma(1+i)\alpha_h\lambda_1' g_1(g_2'-c_2')}{D} \simeq -D,
\end{aligned}$$

where

$$\begin{aligned}
D = &-\alpha_h\lambda_1'H'\left(u\right)\left\{\gamma\alpha_h\lambda_2'\left[\left(1-\omega\right)g_2 - \omega\sigma_s g_1\right] + \left(1-\omega\right)\gamma\alpha_h\lambda_2 g_2' - \left(1-\omega\right)ig_2'\right\}\\
&-\left(1-\omega\right)\gamma\alpha_h\alpha_h'\lambda_1'\lambda_2(g_2'-c_2')\left[\left(1-\omega\right)g_2 - \omega\sigma_s g_1\right]\\
&+\omega(1-\omega)\alpha_h'\lambda_1(g_2'-c_2')\left[\sigma_s i g_1' + \sigma_\ell\gamma\left(1+i\right)g_1' - \sigma_s\gamma\alpha_h\lambda_2 g_1'\right]\\
&-\omega\alpha_h'\lambda_1(g_1'-c_1')\left\{\gamma\alpha_h\lambda_2'\left[\left(1-\omega\right)g_2 - \omega\sigma_s g_1\right] + \left(1-\omega\right)\gamma\alpha_h\lambda_2 g_2' - \left(1-\omega\right)ig_2'\right\}.
\end{aligned}$$

It remains to find the sign of $D$. We follow the same approach as we used above to reduce the equation system to two equations. In (23), $q^2$ is a function of $\left(q^1, s_\ell\right)$.

Recall that $s_\ell = \gamma i \lambda_2 / \lambda_1$. We can transform (23) to

$$g\left(q^2\right) = \frac{\omega g_1}{1-\omega}\left[\sigma_s + \frac{\sigma_\ell \gamma (1+i)\lambda_1}{i(\lambda_1 - \gamma\lambda_2)}\right]. \tag{31}$$

Now we have two equations (10) and (24) to solve for $(q^1, u)$. In (24), we view $q^2$as a function of $q^1$ implicitly defined in (31), and

$$\frac{dq^2}{du} = \frac{\overbrace{\omega\sigma_s i g_1'(\lambda_1-\gamma\lambda_2) + \omega\sigma_s i\lambda_1' g_1 + \omega\sigma_\ell\gamma(1+i)\lambda_1' g_1 + \omega\sigma_\ell\gamma(1+i)\lambda_1 g_1' - (1-\omega) i\lambda_1' g_2}^{\Pi_1}}{\underbrace{(1-\omega) i g_2'(\lambda_1-\gamma\lambda_2) - (1-\omega)\gamma i\lambda_2' g_2 + \omega\sigma_s\gamma i\lambda_2' g_1}_{\Pi_2}}\frac{dq^1}{du}.$$

We can verify that $\Pi_1 > 0$ and $\Pi_2 > 0$. Substitute $dq^2/du$ into (28),

$$\frac{dq^1}{du} = \frac{H'(u)\Pi_2}{\omega(g_1' - c_1')\Pi_2 + (1-\omega)(g_2' - c_2')\Pi_1} < 0.$$

It implies that in the $(u, q^1)$ space, (24) is downward sloping. Moreover, when $u \to 0$, $q^1$ should approach infinity. As before, (10) is also downward sloping in the $(u, q^1)$ space and $q^1$ is finite when $u \to 0$. The intersection of (10) and (24) gives equilibrium $(q^1, u)$. If monetary equilibrium exists and is unique (or we focus on the equilibrium with the smallest $q^1$), it must be the case that (24) is steeper than (10) at the equilibrium allocation. Mathematically,

$$\frac{H'(u)\Pi_2}{\omega(g_1' - c_1')\Pi_2 + (1-\omega)(g_2' - c_2')\Pi_1} < -\frac{\alpha_h'\lambda_1}{\alpha_h\lambda_1'}. \tag{32}$$

After some algebra, we can show that (32) exactly implies that $D > 0$. Again, $D > 0$ implies that the numerator in $\partial q^2/\partial\sigma_\ell$ is positive. We conclude that

$$\begin{aligned}
&\frac{\partial q^1}{\partial \sigma_s} > 0,\ \frac{\partial q^2}{\partial \sigma_s} > 0,\ \text{and}\ \frac{\partial u}{\partial \sigma_s} < 0;\\
&\frac{\partial q^1}{\partial \sigma_\ell} > 0,\ \frac{\partial q^2}{\partial \sigma_\ell} > 0,\ \text{and}\ \frac{\partial u}{\partial \sigma_\ell} < 0.
\end{aligned}$$

# D Appendix D: Equilibrium Existence Conditions

In the extension, different types of monetary equilibrium exist depending on the values of $(i, \sigma_s, \sigma_\ell)$. We have the following nine cases.

(1) When $i = 0$, $\sigma_s \geq 0$ and $\sigma_\ell \geq 0$, we have the Friedman rule equilibrium. In this case, $i_s = i_\ell = i = 0$.

(2) When $i > 0$, $\sigma_\ell = 0$ and $0 \leq \sigma_s \leq (1-\omega)/\omega$, we have an equilibrium that resembles the liquidity trap equilibrium in the basic model. In this case, $i_s = i_\ell = 0$.

(3) When $i > 0$, $\sigma_\ell = 0$ and $(1-\omega)/\omega < \sigma_s < (1-\omega) g(q^2)/[\omega g(q^1)]$ where $(q^1, q^2)$ are solved from $\alpha_h(u)\lambda(q^1) = i, \alpha_h(u)\lambda(q^2) = 0$ and (24), we have an equilibrium that resembles the scarce bonds equilibrium in the basic model. In this case, $0 = i_\ell < i_s < i$.

(4) When $i > 0$, $\sigma_\ell = 0$ and $\sigma_s \geq (1-\omega) g(q^2)/[\omega g(q^1)]$ where $(q^1, q^2)$ are solved from $\alpha_h(u)\lambda(q^1) = i, \alpha_h(u)\lambda(q^2) = 0$ and (24), we have an equilibrium that resembles the plentiful bonds equilibrium in the basic model. In this case, $0 = i_\ell < i_s = i$.

(5) When $i > 0$, $\sigma_s = 0$ and $(1-\omega)(1-\gamma) i/[\omega\gamma(1+i)] < \sigma_\ell$

$< (1-\omega) i g(q^2)/[\omega\gamma(1+i) g(q^1)]$, where $(q^1, q^2)$ are solved from $\alpha_h(u)\lambda(q^1) = i, \gamma\alpha_h(u)\lambda(q^2) = 0$ and (24), we have a scarce bonds equilibrium where only long-term bonds are held by households. In this case, $0 = i_s < i_\ell < i$. Here, it also must be true that $i_\ell > (1-\gamma) i/(1+\gamma i)$ to ensure that long-term bonds are valued by households. Notice that this is the case we discuss in Proposition 3.

(6) When $i > 0$, $\sigma_s = 0$ and $\sigma_\ell \geq (1-\omega) i g(q^2)/[\omega\gamma(1+i) g(q^1)]$ where $(q^1, q^2)$ are solved from $\alpha_h(u)\lambda(q^1) = i, \alpha_h(u)\lambda(q^2) = 0$ and (24), we have a plentiful equilibrium where only long-term bonds are held by households. In this case, $0 = i_s < i_\ell = i$.

(7) When $i > 0$, $\sigma_s \geq 0$, $\sigma_\ell \geq 0$ and $\sigma_s + \gamma(1+i)\sigma_\ell/[(1-\gamma) i] \leq (1-\omega)/\omega$, we have a liquidity trap equilibrium where both bonds have the same return as money.

In this case $0 = i_s < i_\ell = (1-\gamma)\, i / (1+\gamma i) < i$.

(8) When $i > 0$, $\sigma_s > 0$, $\sigma_\ell > 0$, $\sigma_s + \gamma (1+i)\, \sigma_\ell / [(1-\gamma)\, i] \geq (1-\omega)/\omega$ and $\sigma_s + \gamma (1+i)\, \sigma_\ell / i \leq (1-\omega)\, g(q^2) / [\omega g(q^1)]$ where $(q^1, q^2)$ are solved from $\alpha_h(u)\, \lambda(q^1) = i, \alpha_h(u)\, \lambda(q^2) = 0$ and (24), we have a scarce bonds equilibrium where both bonds are valued by households. In this case, $0 < i_s < i_\ell < i$. This is the scarce bonds equilibrium that we discuss in Proposition 4.

(9) When $i > 0$, $\sigma_s \geq 0$, $\sigma_\ell \geq 0$ (with at least $\sigma_s > 0$ or $\sigma_\ell > 0$) and $\sigma_s + \gamma (1+i)\, \sigma_\ell / i \geq (1-\omega)\, g(q^2) / [\omega g(q^1)]$ where $(q^1, q^2)$ are solved from $\alpha_h(u)\, \lambda(q^1) = i, \alpha_h(u)\, \lambda(q^2) = 0$ and (24), we have a plentiful bonds equilibrium where both bonds are valued by households. In this case, $0 < i_s = i_\ell = i$.

If we further classify these equilibrium existence conditions, we have six cases.

(1)When $i = 0$, $\sigma_s \geq 0$ and $\sigma_\ell \geq 0$, we have the Friedman rule equilibrium. In this case, $i_s = i_\ell = 0$.

(2) When $i < 0$, $\sigma_s \geq 0$, $\sigma_\ell \geq 0$ and $\sigma_s + \gamma (1+i)\, \sigma_\ell / [(1-\gamma)\, i] \leq (1-\omega)/\omega$, we have the liquidity trap equilibrium. In this case, $0 = i_s \leq i_\ell \leq (1-\gamma)\, i / (1+\gamma i) < i$.

(3) When $i > 0$, $\sigma_\ell = 0$ and $(1-\omega)/\omega < \sigma_s < (1-\omega)\, g(q^2) / [\omega g(q^1)]$ where $(q^1, q^2)$ are solved from $\alpha_h(u)\, \lambda(q^1) = i, \alpha_h(u)\, \lambda(q^2) = 0$ and (24), we have an equilibrium that resembles the scarce bonds equilibrium in the basic model. In this case, $0 = i_\ell < i_s < i$.

(4) When $i > 0$, $\sigma_s = 0$ and $(1-\omega)(1-\gamma)\, i / [\omega\gamma (1+i)] < \sigma_\ell$

$< (1-\omega)\, i g(q^2) / [\omega\gamma (1+i)\, g(q^1)]$, where $(q^1, q^2)$ are solved from $\alpha_h(u)\, \lambda(q^1) = i, \alpha_h(u)\, \lambda(q^2) = 0$ and (24), we have a scarce bonds equilibrium where only long-term bonds are held by households. In this case, $0 = i_s < i_\ell < i$. Here, it also must be true that $i_\ell > (1-\gamma)\, i / (1+\gamma i)$ to ensure that long-term bonds are valued by households. Notice that this is the case that we discuss in Proposition 3.

(5) When $i > 0$, $\sigma_s > 0$, $\sigma_\ell > 0$, $\sigma_s + \gamma (1+i)\, \sigma_\ell / [(1-\gamma)\, i] \geq (1-\omega)/\omega$ and $\sigma_s + \gamma (1+i)\, \sigma_\ell / i \leq (1-\omega)\, g(q^2) / [\omega g(q^1)]$, we have a scarce bonds equilibrium

where both bonds are valued by households. In this case, $0 < i_s < i_\ell < i$. This is the scarce bonds equilibrium that we discuss in Proposition 4.

(6) When $i > 0$, $\sigma_s \geq 0$, $\sigma_\ell \geq 0$ (with at least $\sigma_s > 0$ or $\sigma_\ell > 0$) and $\sigma_s + \gamma\,(1+i)\,\sigma_\ell/i \geq (1-\omega)\,g\,(q^2)\,/\,[\omega g\,(q^1)]$ where $(q^1, q^2)$ are solved from $\alpha_h\,(u)\,\lambda\,(q^1) = i, \alpha_h\,(u)\,\lambda\,(q^2) = 0$ and (24), we have the plentiful bonds equilibrium where either one type of bonds or both bonds are valued by households. In this case, $i_s = i$ if $\sigma_s > 0$ and $i_\ell = i$ if $\sigma_\ell > 0$.

# References


[1] Berentsen, A., Menzio, G. and Wright, R. "Inflation and Unemployment in the Long Run," *American Economic Review* 101 (2011), 371-98.

[2] Bethune, Z., Rocheteau, G. and Rupert, P. "Unemployment and Household Unsecured Debt," *Review of Economic Dynamics* 18 (2015), 77-100.

[3] D'Amico, S., English, W., Lopez-Salido, D., and Nelson, E., "The Federal Reserve's Large-scale Asset Purchase Programs: Rational and Effects," *The Economic Journal* 122 (2012), 415-46.

[4] Geromichalos, A., Herrenbrueck, L. and Salyer, K., "A Search-Theoretic Model of the Term Premium," Working Papers 138, Department of Economics, University of California, Davis, 2013

[5] Herrenbrueck, L., "Quantitative Easing and the Liquidity Channel of Monetary Policy," working paper, Department of Economics, Simon Fraser University, 2014.

[6] Kalai, E., "Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons," *Econometrica* 45 (1977), 1623-30.

[7] Keys, B.J., Piskorski, T., Seru, A. and Yao, V., "Mortgage Rates, Household Balance Sheets, and the Real Economy", NBER Working Paper No. 20561, 2014

[8] Lagos, R. and Wright, R., "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy* 113 (2005), 463-84.

[9] Maggio, M.D., Kermani, A. and Ramcharan, R., "Monetary Policy Pass-Through: Household Consumption and Voluntary Deleveraging", mimeo, 2015

[10] Mahmoudi, B., "Liquidity Effects of Central Banks' Asset Purchase Programs," MPRA Paper No. 49424, 2013

[11] Mian, A., Rao, K. and Sufi, A., "Household Balance Sheets, Consumption, and the Economic Slump," *The Quarterly Journal of Economics* 128 (2013), 1687-1726.

[12] Mian, A. and Sufi, A., "What Explains the 2007–2009 Drop in Employment?" *Econometrica* 82 (2014), 2197–223.

[13] Mortensen, D. and Pissarides, C., "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies* 61 (1994), 397-415.

[14] Nosal, E. and Rocheteau, G., "Pairwise Trade, Asset Prices, and Monetary policy," *Journal of Economic Dynamics and Control* 37 (2013), 1-17.

[15] Rocheteau, G. and Rodriguez-lopez, A., "Liquidity Provision, Interest Rates, and Unemployment," *Journal of Monetary Economics* 65 (2014), 80-101.

[16] Rocheteau, G., Wright, R. and Xiao, S.X., "Open Market Operations," mimeo, 2015

[17] Silveira, R. and Wright, R., "Search and the Market for Ideas, " *Journal of Economic Theory* 145 (2010), 1550-73.

[18] Wen, Y., "Evaluating Unconventional Monetary Policies—Why Aren't They More Effective," FRB of St. Louis Working Paper 2013-028B, 2013

[19] Williamson, S., "Liquidity, Monetary Policy, and the Financial Crisis: a New Monetarist approach," *American Economic Review*, 102 (2012): 2570-605.

[20] Williamson, S., "Scarce Collateral, the Term Premium, and Quantitative Easing". Working Paper, 2013