Open Market Operations*

Guillaume Rocheteau

University of California - Irvine

Randall Wright

UW - Madison, FRB Minneapolis, FRB Chicago and NBER

Sylvia Xiaolin Xiao

University of Technology, Sydney

November 12, 2014

Abstract

Standard monetary models are extended to incorporate, in addition to currency, liquid government bonds. We then study the impact of policy, including OMO's (open market operations), under various specifications for market structure and for the liquidity of money and bonds – i.e., their acceptability or pledgeability as media of exchange or collateral. Theory delivers sharp predictions for the effects of policy, and generates novel phenomena, like the possibility of negative nominal interest rates, endogenous market segmentation, endogenous price sluggishness, and liquidity traps. We also explain differences in asset liquidity (acceptability or pledgeability) using information theory.

*We thank Aleks Berentsen, Ricardo Cavalcanti, Jonathan Chiu, Mei Dong, Benoit Julien, Ricardo Lagos, Han Han, Chris Waller, Steve Williamson and Yu Zhu for input on this and related work. Wright acknowledges support from the Ray Zemon Chair in Liquid Assets at the Wisconsin School of Business. The usual disclaimer applies.

"Look at Rothschild – you'd think he was selling apples instead of government bonds." *The House of Rothschild* (1934 film)

1 Introduction

Monetary policy is believed by many to have important effects on the economy. Broadly speaking, governments issue two categories of paper, fiat currency and bonds, although there are subcategories, including currencies of different denominations and real or nominal bonds with different maturities. Monetary policy consists of controlling the amounts of these objects outstanding, or their growth rates, often in an attempt to target some nominal interest or inflation rate. There are different ways to change the supply of government-issued assets held by the public, including transfers and spending on goods or other assets. The conventional policy used to alter the mix is to buy or sell bonds for cash – an *open market operation*, or OMO. This project studies the effects of these kinds of policies through the lens of the New Monetarist framework.[1]

In this framework, at some points in time agents trade with each other in decentralized markets, as in search theory, while at other points in time they trade in more centralized markets, as in general equilibrium theory. When they trade with each other, frictions in the environment make it interesting to ask *how* they trade: Do they use barter, credit or media of exchange? If they use credit, is it unsecured or secured? What assets serve as media of exchange or constitute acceptable collateral? We spend some time analyzing why different assets, like currency or bonds, may be more or less acceptable as media of exchange or pledgeable as collateral – i.e., why they may be more or less *liquid*. Both for cases where liquidity is exogenous and where it is derived endogenously, we analyze

[1] Recent expositions of this literature include Williamson and Wright (2010*a*,*b*), Wallace (2010), Nosal and Rocheteau (2011) and Lagos et al. (2014). Our use of the approach here means that we do not impose sticky nominal prices. While New Monetarist models can accommodate and even endogenize sticky prices, as discussed in these references, there is a belief that one does not need *ad hoc* nominal rigidities for interesting policy analyses.

what happens when monetary policy changes under various scenarios for market structure, including random or directed search, and bargaining or posting.

One policy instrument is the money supply, the growth rate of which equals inflation in equilibrium. While the growth rate matters, as in most models, with flexible prices, the level of the money supply does not. Hence, OMO's are effectively the same as changing the stock of outstanding bonds.[2] Theory delivers sharp predictions for these effects, and generates novel phenomena like *negative nominal interest rates*, endogenous *market segmentation*, outcomes resembling *liquidity traps*, and the appearance of *sluggish nominal prices*. For some parameters, injecting cash by buying bonds reduces nominal bond returns and stimulates output through a straightforward but previously neglected channel. Importantly, the effects do not follow from expanding the money supply, but from contracting the bond supply. Sluggish prices emerge as follows: First, increasing the money supply has the direct effect of lowering the value of money so that real balances remain constant (classical neutrality). But having fewer bonds makes agents want to substitute into other forms of liquidity – here, cash – which raises equilibrium real balances, so the value of money falls by less than the increase in the supply. This certainly looks like sluggish nominal adjustment, although by design here sticky prices *per se* have no implications for welfare or policy.

For other parameters, the economy can fall into a trap where output is very low, OMO's cannot affect the situation, and nominal bond rates freeze at a lower bound that may or may not be zero. While a lower bound of zero almost always appears in theory, negative nominal returns do occur in practice.[3] Although negative nominal rates are notoriously hard to capture using standard macro models, assets with negative nominal yields should be no surprise when those assets pro-

[2] In this respect our analysis is similar to recent papers by Williamson (2012,2014*a*,*b*) and Han (2014), and earlier work by Geromichalos et al. (2007) and Lagos and Rocheteau (2008).

[3] See *Wall Street Journal* (Aug. 10, 2012) or *The Economist* (July 14, 2014) for discussion. At the time of this writing, the nominal yield on T-bills hovers around 0 in the US, the current interest rate on reserves at the ECB is -0.1%, nominal German bond yields are negative out to 3 years, and a similar situation exists in Switzerland, as discussed in more detail below.

vide services in addition to returns. As a leading example, with travellers' checks the service is insurance against loss or theft. Here it is liquidity, not insurance, that is center stage, but the general idea is that once one explicitly models the role of assets in transactions it is not hard to get negative nominal rates. As regards liquidity traps, although discussions can be ambiguous, if not downright mysterious to those less comfortable with IS-LM models, Wikipedia is as good a source as any when it describes situations where "injections of cash ... by a central bank fail to decrease interest rates and hence make monetary policy ineffective." Such situations can arise in the models here for reasons that are fully compatible with fundamental microeconomics.

There is a literature on monetary policy with market segmentation, including Alvarez et al. (2001,2002,2009). These papers use CIA (cash-in-advance) models. Sometimes they assume only a fraction of agents can participate in bond markets, making OMO's affect interest rates due to so-called liquidity effects. Or they assume a fixed cost to using bonds in goods markets. See Kahn (2006) for a broader review. Here segmentation is in terms of the assets that are accepted in different submarkets. Our agents can freely participate in any submarket, and thus face no CIA constraint in the sense that they can always choose a market where cash is not necessary. In equilibrium, they hold different portfolios, depending on the submarket they choose, but these differences are neither due to transaction costs nor exogenous restrictions on asset usage or market participation. In fact, the results have more to do with *market tightness* as usually modeled using search theory. We regard this framework as complementary to the existing literature, trying to understand related phenomena using a different approach.

Although we use methods from search theory, it is well understood in modern monetary economics (see fn. 1) that search *per se* is not an essential ingredient; it is used here for convenience, and because it seems a natural way to think about market segmentation where buyers choose to visit sellers that accept particular payment instruments. Of course, it is important to ask *why* sellers treat different assets differently. We follow a body of work (references given below) that has some agents less able to discern legitimate from fraudulent versions of certain assets. This means the agents may reject those assets outright, or accept them only up to endogenous thresholds. Now liquidity is not invariant to changes in policy, and it is easy to get multiple equilibria; still, theory delivers sharp predictions about the effects of inflation and OMO's. Taken together, we think the results provide a rigorous treatment of the effects of monetary policy, where the economic channels are novel, yet transparent.

The rest of the paper is organized as follows. Section 2 describes the baseline environment. Section 3 considers equilibrium with random matching and bargaining, discusses negative nominal yields, sluggish prices and liquidity traps, taking as given asset acceptability and pledgeability. Section 4 endogenizes acceptability and pledgeability based on asymmetric information. Section 5 considers directed search and endogenous segmentation. Section 6 concludes.

2 Environment

Time is discrete and continues forever. In each period two markets convene sequentially: first there is a decentralized market, or DM, with frictions detailed below; then there is a frictionless centralized market, or CM. Each period in the CM, a large number of infinitely-lived agents work, consume and adjust their portfolios. In the DM, some of these agents, called *sellers* and indexed by s, can produce a good different from the CM good, but do not want to consume, while others, called *buyers* and indexed by b, want to consume this good but cannot produce. Generally, μ is the measure of buyers, and n is the ratio of the sellers to buyers in the DM, where they meet pairwise, with α denoting the probability a buyer meets a seller and α/n the probability a seller meets a buyer.

The period payoffs for buyers and sellers are

$$\mathcal{U}^b(q,x,\ell) = u(q) + U(x) - \ell \text{ and } \mathcal{U}^s(q,x,\ell) = -c(q) + U(x) - \ell, \tag{1}$$

where q is the DM good, x is the CM good and numeraire, and ℓ is labor supply. For sellers, $c(q)$ is a cost of production. For buyers, one can interpret $u(q)$ as a utility function, or as a production function taking q as an input and delivering $x = u(q)$ as output that enters the next CM budget equation, given that CM payoffs are linear in numeraire, as shown below. The same equations can therefore represent DM transactions as consumers acquiring output q, or producers investing in input q, which is relevant to the extent that liquidity considerations impinge on both households and firms.[4]

As usual, U, u and c are twice continuously differentiable with $U' > 0$, $u' > 0$, $c' > 0$, $U'' < 0$, $u'' < 0$ and $c'' \geq 0$. Also, $u(0) = c(0) = 0$, and there is a $\hat{q} > 0$ such that $u(\hat{q}) = c(\hat{q}) > 0$. Define the efficient q by $u'(q^*) = c'(q^*)$. Quasi-linearity in (1) simplifies the analysis because it leads to a degenerate distribution of assets across agents *of a given type* at the start of each DM, and because it makes CM payoffs linear in wealth.[5] There is a discount factor $\beta = 1/(1+r)$, $r > 0$, between the CM and DM, while any discounting between the DM and CM can be subsumed in the notation in (1). It is assumed that x and q are nonstorable, to rule out direct barter, and that agents are to some degree anonymous in the DM, to hinder unsecured credit. This is what generates a role for assets in the facilitation of intertemporal exchange.

[4]To think about it as a market for reallocating investment goods, imagine everyone the DM is a producer with capital k. They realize a shock making their production function either $f(k)$ or $g(k)$ with $g'(k) > f'(k)$. In meetings between low- and high-productivity producers, if the former gives q units of k to the latter, we can write $u(q) = g(q+k) - g(k)$ and $c(q) = f(k) - f(k-q)$ and keep everything else in the model the same.

[5]Wong (2012) shows these same results obtain for any $\tilde{U}(x, 1-\ell)$ with $\tilde{U}_{11}\tilde{U}_{22} = \tilde{U}_{12}^2$, which holds for quasi-linear utility, but also any $\tilde{U}$ that is homogeneous of degree 1, including $\tilde{U} = x^a(1-\ell)^{1-a}$ or $\tilde{U} = [x^a + (1-\ell)^a]^{1/a}$. Alternatively, Rocheteau et al. (2007) show these same results obain for any $\tilde{U}$ if we assume indivisible labor à la Rogerson (1988). In any case, the implicit constraints $x \geq 0$, $q \geq 0$ and $\ell \in [0,1]$ are assumed slack.

There are two assets that can potentially serve in this capacity: money; and government bonds called T-bills. Their supplies are A_m and A_b. Their CM prices are ϕ_m and ϕ_b. The benchmark specification has short-term real bonds that, somewhat like *Arrow securities*, are issued in one CM and pay 1 unit of numeraire in the next, although we show below how to accommodate nominal or long-term bonds. The real value of money and bonds per buyer are z_m and z_b. For money, $z_m = \phi_m A_m$; for real bonds, $z_b = A_b$; for nominal bonds as discussed below $z_b = \phi_m A_b$; and for long-term bonds $z_b = (\phi_b + \gamma) A_b$ where γ is a real coupon paid each CM. In any case, these assets are *partially liquid*, in the sense that they are accepted in at least some DM transactions at least up to some limit. There are two standard interpretations. Following Kiyotaki and Wright (1989,1993), sellers may only accept some assets as media of exchange (immediate settlement). Following Kiyotaki and Moore (1997,2005), they may only accept some assets as collateral securing promises of numeraire in the next CM (deferred settlement), with the idea being that those who renege on promises are punished by having some assets seized. Our results apply to either interpretation.

There is a third interpretation in terms of repos (repurchase agreements): buyers in the DM give assets to sellers, who give them back in the CM at a prearranged price. Of course, in theory, it is not necessary for agents to get back the same assets, when they are fungible, or to prearrange a price, when they trade in frictionless markets. Still, as in actual repo practice, assets in the model aid in intertemporal trade. One advantage of repos here is that one may not worry as much about certain details concerning secured lending – e.g., exactly who does the record keeping and seizes debtors' assets if they default? In any event, we prefer to be agnostic about some institutional niceties for the purpose of this exercise. To be clear, the point is not that there is anything 'deep' about this discussion of settlement, collateral and repos; it is rather that different interpretations apply more or less interchangeably to the same formalization.

With each of these interpretations it is assumed that only a fraction $\chi_j \in [0,1]$ of asset j can be used in the DM, either as a payment instrument or collateral. Unless otherwise indicated, $\chi_m > 0$ so currency can be valued, and $\chi_b > 0$ so the model does not reduce to a pure-currency economy as in Lagos and Wright (2005). Under the deferred settlement interpretation, these pledgeability parameters describe the haircut one takes when using z_j as collateral, often motivated by saying debtors can abscond with a fraction $1 - \chi_j$ of an asset after default. In Section 4 we show how to endogenize χ_j whether z_j is used for immediate or deferred settlement when counterfeiting is a potential issue, and note that if we interpret money broadly to include demand deposits, counterfeits include bad checks. While $\chi_j < 1$ is not critical, and most results go through with $\chi_j = 1$, there is no reason to impose that restriction at this point.

In the DM, α_m is the probability a buyer meets a seller that accepts only money; α_b is the probability he meets one that accepts only bonds; and α_2 is the probability he meets one that accepts both. Below we interpret $\alpha_j = \alpha n_j$ as the product of a underlying arrival rate α and the fraction n_j of type-j sellers. Special cases include ones where everyone accepts cash, $\alpha_b = 0$; the assets are perfect substitutes, $\alpha_b = \alpha_m = 0$; and a pure-currency economy, $\alpha_b = \alpha_2 = 0$. Under the deferred settlement interpretation, since agents renege iff debt exceeds the value of collateral, trades are constrained by asset holdings, as in immediate settlement. To streamline the presentation we usually frame the discussion in terms of media of exchange, but it is good to keep in mind that it is basically a relabeling to switch between Kiyotaki-Wright money and Kiyotaki-Moore credit. In particular, to motivate $\alpha_b > 0$, note that while few retailers take T-bills and not currency, financial institutions regularly use bonds as collateral. While $\alpha_b > 0$, like $\chi_j < 1$, is not critical, there is no reason to impose $\alpha_b = 0$ at this point.

We focus on stationarity equilibria, where $z_m = \phi_m A_m$ is constant, so the money growth rate π equals the inflation rate: $\phi_m/\phi_{m,+1} = A_{m,+1}/A_m = 1 + \pi$, where subscript $+1$ indicates next period. Stationarity also entails z_b constant, which means A_b is constant for real bonds, while the ratio $B = A_b/A_m$ is constant for nominal bonds. We restrict attention to $\pi > \beta - 1$, or the limit $\pi \to \beta - 1$, which is the Friedman rule (there is no monetary equilibrium with $\pi < \beta - 1$). The government's budget constraint is

$$G + T - \pi\phi_m A_m + A_b(1 - \phi_b) = 0, \tag{2}$$

where the first term is their consumption of x, the second is a lump-sum transfer, the third is seigniorage, and the fourth is debt service, assuming short-term real bonds, with an obvious adjustment in the case of nominal or long-term bonds. Given the other variables, we assume T adjusts to satisfy (2) each period.

It is crucial to distinguish between different interest rates. Define the return on an illiquid nominal bond – one that is never accepted in the DM – by the Fisher equation, $1 + \iota = (1 + \pi)/\beta$, where $1/\beta = 1 + r$ is the return on an illiquid real bond. Thus, $1 + \iota$ is the amount of cash you would need in the next CM to make you indifferent to giving up a dollar today, while $1 + r$ is the amount of x you would need in the next CM to make you indifferent to giving up a unit today. As usual, whether or not these trades are actually made, we can price them, and the Fisher equation must hold as a no-arbitrage condition for bonds that are illiquid in the sense used here. This is not necessarily true for our liquid government bonds, which have a nominal yield denoted ρ. For a real bond, the nominal return is the amount of cash you can get in the next CM by investing a dollar today, $1 + \rho = \phi_m/\phi_b\phi_{m,+1} = (1 + \pi)/\phi_b$ (by way of comparison, for a nominal bond, $1 + \rho = \phi_m/\phi_b$). As in Silveira and Wright (2010) or Rocheteau and Rodriguez-Lopez (2013), it is also convenient to define the *spread* by $s = (\iota - \rho)/(1 + \rho)$.

The Fisher equation implies that the Friedman rule is $\iota = 0$, and that we can use either π or ι as a policy instrument. We typically use ι, but one can always take this to be simply short-hand notation for inflation, since $1+\pi = \beta(1+\iota)$. In terms of economics, ι is the cost of the liquidity services provided by z_m, because rather than holding cash one could made an illiquid investment. Similarly, s is the cost of the liquidity services provided by z_b, because the liquid bond yields ρ while the illiquid bond yields ι. One way to see the connection is to write the Fisher equation and definition of spread as

$$1+\iota = (1+\pi)(1+r) \tag{3}$$

$$1+\iota = (1+\rho)(1+s). \tag{4}$$

3 Random Matching

Our first specification involves random search and bargaining, because it is easy, and common in the related literature. We start with short-term real bonds, then consider versions with other forms of government debt.

3.1 Baseline: Short Real Bonds

A buyer's DM state is his portfolio (z_m, z_b), while in the CM all that matters is the sum $z = z_m + z_b$. Let the CM and DM value functions be denoted $W(z)$ and $V(z_m, z_b)$. Then

$$W(z) = \max_{x,\ell,\hat{z}_m,\hat{z}_b} \{U(x) - \ell + \beta V(\hat{z}_m, \hat{z}_b)\} \text{ st } x = z + \ell + T - (1+\pi)\hat{z}_m - \phi_b \hat{z}_b$$

where $\hat{z}_j$ is the real value of asset j taken out of the CM, and the real wage is $\omega = 1$ because we assume that 1 unit of ℓ produces 1 unit of x. The FOC's for assets are $1+\pi = \beta V_1(\hat{z}_m, \hat{z}_b)$ and $\phi_b = \beta V_2(\hat{z}_m, \hat{z}_b)$. Also, $W'(z) = 1$.[6]

[6] Earlier we alluded to the idea that buyers' DM payoff $u(q)$ can be interpreted as either the utility of consuming q, or the output of using it to produce numeraire for the next CM, which is valid because $W'(z) = 1$. It may be interesting to expand on this by having the output of q also depend on CM labor, and thus endogenize ω and ℓ. This would generate an inflation-employment tradeoff for reasons related to, but different than, models by Rocheteau et al. (2008), Dong (2011), Aruoba et al. (2011), Berentsen et al. (2011) or Dong and Xiao (2014).

There is a similar CM problem for sellers, but we can assume wlog they carry no assets into the DM: if assets are priced fundamentally they are indifferent to holding them; if assets bear a liquidity premium they strictly prefer not holding them. So, to continue with buyers, in the DM

$$V(z_m, z_b) = W(z_m + z_b) + \alpha_m[u(q_m) - p_m] + \alpha_b[u(q_b) - p_b] + \alpha_2[u(q_2) - p_2]$$

where p_j are payments in type-j meetings and we use $W'(z) = 1$. Payments are constrained by $p_j \leq \bar{p}_j$, where $\bar{p}_j$ is the buyer's *liquidity position* in a type-j meeting: $\bar{p}_m = \chi_m z_m$, $\bar{p}_b = \chi_b z_b$ and $\bar{p}_2 = \chi_m z_m + \chi_b z_b$. For now, χ_j is exogenous; it is endogenized in Section 4.[7]

The terms of trade are determined for now by bargaining: to get q you pay $p = v(q)$, where $v(\cdot)$ depends on the solution concept. Kalai's (1977) proportional solution, e.g., implies $v(q) = \theta c(q) + (1-\theta) u(q)$, where θ is buyers' bargaining power. However, other than $v(0) = 0$ and $v'(q) > 0$, for now all we need is this: Let $p^* = v(q^*)$ be the payment required to get the efficient amount. Then $p^* \leq \bar{p}_j \implies p_j = p^*$ and $q_j = q^*$, while $p^* > \bar{p}_j \implies p_j = \bar{p}_j$ and $q_j = v^{-1}(\bar{p}_j)$. This holds for Kalai and Nash bargaining, although we prefer the former, as it has several attractive properties relative to the latter in these kinds of models (Aruoba et al. 2007). It also holds for creatively designed bilateral trading mechanisms (Hu et al. 2009; Gu et al. 2014), and for Walrasian pricing (Rocheteau and Wright 2005) or auctions (Galenianos and Kircher 2008) when agents in the DM trade multilaterally. In any case, for now all we use is $p = v(q)$.

As is standard, $\iota > 0$ implies buyers pay all they can in type-m meetings and and are still constrained: $p_m = \chi_m z_m < p^*$. Since $\chi_m z_m < p^*$, in type-2 meetings buyers may as well pay all they can in cash before using bonds, because in these

[7] Also, under the interpretation of p as a promise to pay in the CM, we can add unsecured debt up to a limit $\bar{d}$, which may be exogenous, or endogenous as in Kehoe and Levine (1993). Then the bound on p_m, e.g., becomes $\bar{p}_m = \chi_m z_m + \bar{d}$, but this does *not* affect the outcome. As in Gu et al. (2014), one can show that if $\bar{d}$ changes z_m responds endogenously to keep $\bar{p}_m$ the same. Hence, at least if $\alpha_b = 0$, $\bar{d} = 0$ is wlog.

meetings agents are indifferent to any combination of z_m and z_b. Buyers use all the bonds they can in type-2 meetings iff $\bar{p}_2 \leq p^*$, and use all they bonds they can in type-b meetings iff $\bar{p}_b \leq p^*$. It is clear that $p_2 \geq p_b$. What must be determined is whether: 1. $p_2 = \bar{p}_2$ and $p_b = \bar{p}_b$ (buyers are constrained in all meetings); 2. $p_2 < \bar{p}_2$ and $p_b = \bar{p}_b$ (they are constrained in type-b but not type-2 meetings); or 3. $p_2 < \bar{p}_2$ and $p_b < \bar{p}_b$ (they are unconstrained in both). We consider each case in turn, assuming throughout that a monetary equilibrium exists.[8]

Case 1 is the most interesting, with buyers constrained in all meetings:

$$v(q_m) = \chi_m z_m,\ v(q_b) = \chi_b z_b, \text{ and } v(q_2) = \chi_m z_m + \chi_b z_b. \tag{5}$$

The Euler equations are derived by differentiating $V(z_m, z_b)$ using (5) and inserting the results into the FOC's for assets in the CM. The results are

$$1 + \pi = \beta \left[1 + \alpha_m \chi_m \lambda(q_m) + \alpha_2 \chi_m \lambda(q_2)\right] \tag{6}$$

$$\phi_b = \beta \left[1 + \alpha_b \chi_b \lambda(q_b) + \alpha_2 \chi_b \lambda(q_2)\right], \tag{7}$$

where $\lambda(q_j) \equiv u'(q_j)/v'(q_j) - 1$ is the *liquidity premium* in a type-j meeting, i.e., the Lagrange multiplier on $p_j \leq \bar{p}_j$. If $\chi_b = 0$, bonds would be priced *fundamentally* at $\phi_b = 1/\beta$. In any case, (6)-(7) simplify to

$$\iota = \alpha_m \chi_m \lambda(q_m) + \alpha_2 \chi_m \lambda(q_2) \tag{8}$$

$$s = \alpha_b \chi_b \lambda(q_b) + \alpha_2 \chi_b \lambda(q_2), \tag{9}$$

using the nominal rate on illiquid bonds ι and the spread s defined above.[9]

[8] It is routine to show $\alpha_m > 0$ implies monetary equilibrium exists iff $\iota < \bar{\iota}_m$, while $\alpha_m = 0$ implies it exists iff $\alpha_2 > 0$, $\chi_b A_b < p^*$ and $\iota < \bar{\iota}_2$, where $\bar{\iota}_m$ and $\bar{\iota}_2$ may or may not be finite.

[9] Notice (9) is reminiscent of the *convenience yield* notion in Krishnamurphy and Vissing-Jorgenson (2012), measured by the difference between government and corporate bond yields. One could say (9) 'rationalizes' their reduced-form with bonds in the utility function, but that is not our goal, any more than having (8) 'rationalize' money in the utility function. The goal is to study the role of assets in exchange, not to make the mundane point that this role leads to assets appearing in indirect utility functions. Moreover, (8)-(9) suggest estimating the *interrelated asset demand system*, not demand for T-bills or money in isolation.

Given $1+\rho = (1+\pi)/\phi_b$, (6)-(7) immediately imply

$$\rho = \frac{\alpha_m \chi_m \lambda(q_m) - \alpha_b \chi_b \lambda(q_b) + (\chi_m - \chi_b)\alpha_2 \lambda(q_2)}{1 + \alpha_b \chi_b \lambda(q_b) + \alpha_2 \chi_b \lambda(q_2)} \tag{10}$$

Notice $\rho < 0$ is possible, in contrast to conventional theory. There are two ways to get $\rho < 0$: if $\chi_m = \chi_b$ then $\rho < 0$ iff $\alpha_m \lambda(q_m) < \alpha_b \lambda(q_b)$; and if $\alpha_m \lambda(q_m) = \alpha_b \lambda(q_b)$ then $\rho < 0$ iff $\chi_m < \chi_b$ and $\alpha_2 \lambda(q_2) > 0$. Related models by Williamson (2012), Dong and Xiao (2013) or Rocheteau and Rodriguez-Lopez (2013) have $\alpha_b = 0$ and $\chi_b = \chi_m = 1$, making $\rho < 0$ impossible. Whether our generalizations are realistic, they describe logically how to get $\rho < 0$. But according to *The Economist* (July 14, 2014), the logic may well be relevant: "Not all Treasury securities are equal; some are more attractive for repo financing than others. With less liquidity in the market, those desirable Treasuries can be hard to find: some short-term debt can trade on a negative yield because they are so sought after." Consistent with theory, traders in reality want bonds that convey liquidity, and are willing to accept negative nominal yields to get them.[10]

It is important to understand that $\rho < 0$ does not defy standard no-arbitrage conditions, because while individuals can issue bonds –i.e., borrow – they cannot guarantee claims against them are liquid – i.e., circulate in the DM. This is similar to agents accepting negative nominal returns on assets like travellers' checks, or demand deposits, at least if we count fees, that are less susceptible than cash to loss and theft. That does not violate no-arbitrage if agents cannot guarantee the security of their paper in the DM, without incurring some cost, as is presumably incurred with travellers' checks or deposit banking. The model in

[10] Relatedly, consider the informed view of the Swiss National Bank (2013): "With money market rates persistently low and Swiss franc liquidity still high, trading activity on the repo market remained very slight. However, activity on the secured money market did not grind to a complete halt, due to the demand for high-quality securities. The increased importance of these securities is reflected in the trades on the interbank repo market which were concluded at negative repo rates." Aleks Berentsen further suggests to us that Swiss bonds can be used as collateral in markets outside of Switzerland, where francs cannot. So some (mainly foreign) banks use francs on reserve at the central bank to acquire bonds with negative returns to facilitate secured credit, consistent with the model.

He et al. (2008) or Sanches and Williamson (2010) where cash is subject to theft can deliver a negative lower bound for this reason. Somewhat Relatedly, the model in Andolfatto (2013) can deliver a positive lower bound because imperfect commitment/enforcement limits government's ability to tax and deflate. Here it is purely liquidity considerations at work.

A stationary monetary equilibrium is a list (q_m, q_b, q_2, z_m, s) solving (5)-(9) with $z_m > 0$ and $z_b = A_b$. To characterize it, use (5) to rewrite (8) as

$$\iota = \alpha_m \chi_m L(\chi_m z_m) + \alpha_2 \chi_m L(\chi_m z_m + \chi_b z_b), \tag{11}$$

where $L(\cdot) \equiv \lambda \circ v^{-1}(\cdot)$. Given $z_b = A_b$, under standard assumptions (see fn. 8), a solution $z_m > 0$ to (11) exists, is generically unique, and entails $L'(\cdot) < 0$, by the argument in Wright (2010). From z_m, (5) determines (q_m, q_b, q_2). Then (9) determines s, (10) determines ρ, etc. Clearly, a one-time change in A_m is neutral, because ϕ_m adjusts to leave $z_m = \phi_m A_m$ and other real variables the same. Hence, an OMO that swaps A_b for A_m has the same real impact as changing only A_b, assuming changes in the CM price level $1/\phi_m$ are not artificially impeded.

Letting $D_R \equiv \alpha_m \chi_m^2 L'(\chi_m z_m) + \alpha_2 \chi_m^2 L'(\chi_m z_m + \chi_b z_b) < 0$, we have $\partial z_m / \partial \iota = 1/D_R < 0$. As usual, a higher nominal rate on illiquid bonds (or inflation or money growth) reduces real balances. Then

$$\frac{\partial q_m}{\partial \iota} = \frac{\chi_m}{v'(q_m) D_R} < 0, \; \frac{\partial q_b}{\partial \iota} = 0, \text{ and } \frac{\partial q_2}{\partial \iota} = \frac{\chi_m}{v'(q_2) D_R} < 0.$$

In terms of financial variables,

$$\begin{aligned}
\frac{\partial s}{\partial \iota} &= \frac{\alpha_2 \chi_m \chi_b L'(\chi_m z_m + \chi_b z_b)}{D_R} > 0 \\
\frac{\partial \phi_b}{\partial \iota} &= \beta \frac{\alpha_2 \chi_m \chi_b L'(\chi_m z_m + \chi_b z_b)}{D_R} > 0 \\
\frac{\partial \rho}{\partial \iota} &= \frac{\alpha_m L'(\chi_m z_m) + \alpha_2 L'(\chi_m z_m + \chi_b z_b)\left[1 - (1+\rho)\chi_b/\chi_m\right]}{(1+s)\left[\alpha_m L'(\chi_m z_m) + \alpha_2 L'(\chi_m z_m + \chi_b z_b)\right]} \gtrless 0,
\end{aligned}$$

assuming $\alpha_2 > 0$. If $\alpha_2 = 0$ then $\partial s/\partial \iota = \partial \phi_b/\partial \iota = 0$, since $\alpha_2 = 0$ means there is no substitution between z_m and z_b in DM meetings; if $\alpha_2 > 0$, however, then higher ι raises s and ϕ_b as agents try to move out of cash and into bonds.[11]

For OMO's, the effects of changing the stock of outstanding bonds are

$$\begin{aligned}
\frac{\partial z_m}{\partial A_b} &= -\frac{\alpha_2 \chi_m \chi_b L'\left(\chi_m z_m + \chi_b z_b\right)}{D_R} < 0 \\
\frac{\partial q_m}{\partial A_b} &= -\frac{\alpha_2 \chi_b L'\left(\chi_m z_m + \chi_b z_b\right)}{v'\left(q_m\right) D_R} < 0 \\
\frac{\partial q_b}{\partial A_b} &= \frac{\chi_b}{v'\left(q_b\right)} > 0 \\
\frac{\partial q_2}{\partial A_b} &= \frac{\alpha_m \chi_b L'(\chi_m z_m)}{v'\left(q_2\right) D_R} > 0,
\end{aligned}$$

assuming $\alpha_2 > 0$. Higher A_b decreases z_m and q_m because liquidity is less scarce in type-2 meetings, so agents economize on cash, which comes back to haunt them in type-m meetings. Hence OMO's have different effects on q in different meetings, and an ambiguous impact on DM output $\Sigma_j \alpha_j q_j$, either consumption or investment, depending on interpretation. Given $\chi_b > 0$, one can also check $\partial s/\partial A_b < 0$, $\partial \phi_b/\partial A_b < 0$ and $\partial \rho/\partial A_b > 0$.

While Case 1 is the most interesting, for completeness, consider Case 2 with buyers unconstrained in type-2 meetings. The equilibrium conditions are similar except $q_2 = q^*$, so $\lambda\left(q_2\right) = 0$ and z_m is determined as in a pure-currency economy. An increase in ι lowers z_m and q_m, does not affect q_b or q_2, and increases s as agents again try to shift from z_m to z_b. An increase in A_b does not affect z_m, q_m or q_2, increases q_b and decreases s. Finally, consider Case 3 with buyers unconstrained in type-2 and type-b meetings, $q_b = q_2 = q^*$. Now bonds provide no liquidity at the margin, so $s = 0$. Hence, increases in ι reduce z_m and q_m but otherwise affect nothing, while increases in A_b affect nothing.

[11] The only ambiguous effect here is $\partial \rho/\partial \iota$. Intuitively, inflation tends to make the nominal bond return rise via the Fisher (1930) effect, for a given real return, but also tends to make the real return fall via the Mundell (1963) effect. Numerical examples (see Figure 1 below) indicate that ρ can indeed be nonmonotone in ι.

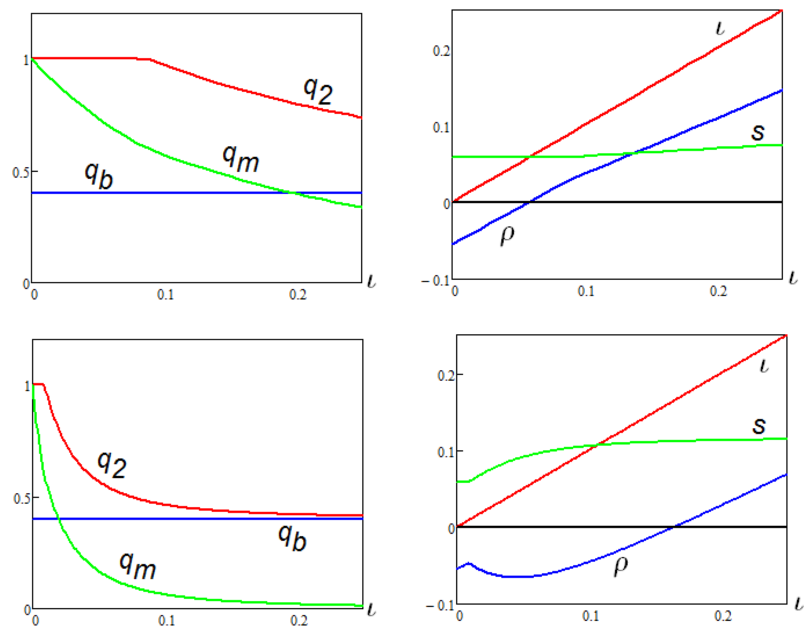

Figure 1: Effects of ι (nominal illiquid interest rate)

Which case obtains? If $\chi_b A_b \geq v(q^*)$ (bonds are abundant) it is Case 3. If $\chi_b A_b < v(q^*)$ (bonds are scarce) it is Case 2 when ι is small and Case 1 when ι is big, as in Figure 1, where Case 1 (Case 2) obtains to the left (right) of the ι at which the curves kink. Effects to notice are: q_m can be above or below q_b; ρ can be negative; and ρ can be nonmonotone in ι. Figure 2 shows the effects of A_b, where Case 1 obtains to the left of the point where q_2 kinks, Case 3 to the right of the point where q_b kinks, and Case 2 in between. While this is not

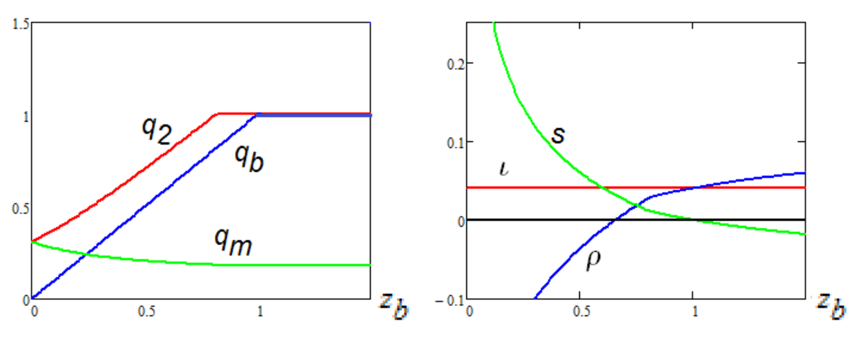

Figure 2: Effects of z_b (supply of liquid bonds)

an empirical paper, we mention that some people argue real-world markets suffer from a scarcity of high-quality liquid assets (BIS 2001; Caballero 2006; Caballero and Krishnamurthy 2006; Gourinchas and Jeanne 2012; IMF 2012; Gorton and Ordonez 2014). In our stylized model this corresponds to Case 1.[12]

Issuing currency and buying bonds – an OMO depicted as a move to the left in Figure 2 – lowers ρ and, at least over some range, increases q_m while it decreases q_2 and/or q_b. We repeat that this has nothing to do with increasing the money supply *per se*, which is neutral, for the classical reason that units do not matter. The real effects are due to decreasing A_b, which stimulates demand for real money balances as an alternative source of liquidity. Thus, an OMO decreases the T-bill rate ρ not by putting more currency in the hands of the public, but by raising the bond price through a contraction in supply. Issuing currency to finance the OMO is irrelevant because, absent *ad hoc* restrictions on the ability of prices to adjust, the direct effect is merely to lower ϕ_m so that z_m stays the same. Note however that due to indirect effects it *looks like* nominal prices are sluggish: in general equilibrium, the net impact of an OMO that injects cash is to raise z_m, which means that the value of money ϕ_m goes down (the price level goes up) by less than the increase in A_m. This is the New Monetarist anatomy of an OMO.[13]

Similar results would emerge if one were to naively insert bonds as generic goods in utility functions, like apples. This is dubbed naive because bonds are not goods, they are assets, valued for their returns plus liquidity services, and the value of liquidity is not a primitive the way the utility of apples might be (e.g.,

[12]Figure 1 uses $u(q) = 2\sqrt{q}$, $c(q) = q$, $v(q) = c(q)$ (buyer-take-all bargaining), $\beta = 0.95$, $A_b = 0.4$, $\alpha_m = 0.3$, $\alpha_2 = 0.2$ and $\alpha_b = 0.1$. The upper panels use $\chi_m = \chi_b = 1$ and the lower panels use $\chi_m = 0.1$ and $\chi_b = 1$. Figure 2 is similar but $\chi_m = \chi_b = 1$ and $i = 0.04$. These are not meant to be realistic, only to illustrate possibilities.

[13]In terms of policy implementation, although we usually take ι as the instrument, we could target the T-bill rate ρ using OMO's or adjustments in ι, which is tied to inflation and hence ultimately to monetary expansion. We can also target multiple variables through combinations of policies. While our setup is stylized, there are related papers delving further into implementation (Berentsen and Monnet 2008; Berentsen and Waller 2011; Afonso and Lagos 2013; Bech and Monnet 2014; Berentsen et al. 2014; Chiu and Monnet 2014). We abstract from these institutional details, while recognizing they are sometimes important.

one ramification is that the value of the liquidity provided by bonds vanishes as $\iota \to 0$). We think it is better to model liquidity explicitly than treat it like fruit juice. Now, some assets are somewhat like apples, like apple trees, and it is only fiat objects that have no intrinsic value. But to the extent that assets provide liquidity, we want to try to take this seriously.

3.2 Variations: Nominal or Long Bonds

Consider now nominal bonds issued in one CM and paying a dollar in the next. For stationarity, let money and bond grow at the same rate π, so $B = A_b/A_m$, $z_m = \phi_m A_m$ and $z_b = Bz_m$ are constant over time. Households' CM budget constraint is now $x = z + \ell + T - (1+\pi)\hat{z}_m - (1+\pi)\hat{z}_b/(1+\rho)$, and there is a similar adjustment for government; otherwise things are similar, and it is straightforward to emulate the method used for real bonds. In Case 1, with buyers always constrained, $v(q_b) = B\chi_b z_m$, $v(q_2) = (\chi_m + B\chi_b) z_m$ and the Euler equation for bonds has a ϕ_m on the RHS, but (8) and (9) are the same.

Letting $D_N \equiv \alpha_m \chi_m^2 L'(\chi_m z_m) + \alpha_2 \chi_m (\chi_m + B\chi_b) L'(\chi_m z_m + B\chi_b z_m) < 0$, we have $\partial z_m/\partial\iota = 1/D_N < 0$. Also,

$$\frac{\partial q_m}{\partial \iota} = \frac{\chi_m}{v'(q_m) D_N} < 0,\ \frac{\partial q_b}{\partial \iota} = \frac{B\chi_b}{v'(q_b) D_N} < 0, \text{ and } \frac{\partial q_2}{\partial \iota} = \frac{\chi_m + B\chi_b}{v'(q_2) D_N} < 0.$$

The only qualitative difference from real bonds is that ι now affects q_b. Moreover,

$$\begin{aligned}
\frac{\partial z_m}{\partial B} &= -\frac{\alpha_2 \chi_m \chi_b z_m L'(\chi_m z_m + B\chi_b z_m)}{D_N} < 0 \\
\frac{\partial q_m}{\partial B} &= -\frac{\alpha_2 \chi_m^2 \chi_b z_m L'(\chi_m z_m + B\chi_b z_m)}{v'(q_m) D_N} < 0 \\
\frac{\partial q_b}{\partial B} &= \frac{\alpha_m \chi_m^2 \chi_b z_m L'(\chi_m z_m) + \alpha_2 \chi_m^2 \chi_b z_m L'(\chi_m z_m + B\chi_b z_m)}{v'(q_b) D_N} > 0 \\
\frac{\partial q_2}{\partial B} &= \frac{\alpha_m \chi_m^2 \chi_b z_m L'(\chi_m z_m)}{v'(q_2) D_N} > 0.
\end{aligned}$$

One can also derive the effects on ρ, s and ϕ_b, study Cases 2 and 3, and delineate parameters for which each obtains. Since the results are qualitatively the same as those in Section 3.1, in what follows we concentrate on real bonds.

Now consider a long-term bond, say a consol that pays in perpetuity γ units of numeraire each CM.[14] For this exercise, purely to reduce notation, we set $\chi_j = 1$. Then given $z_b = (\phi_b + \gamma) A_b$, in Case 1 with buyers always constrained, $v(q_m) = z_m$, $v(q_b) = z_b$ and $v(q_2) = z_m + z_b$. The Euler equation for bonds now has a $\phi_b + \gamma$ on the RHS, but again (8) and (9) are the same, where now the nominal bond yield satisfies $1 + \rho = (1 + \pi)(1 + \gamma/\phi_b)$ and the spread $s = (r\phi_b - \gamma)/(\phi_b + \gamma)$. The main difference from a short-term bond is that $z_b = (\phi_b + \gamma) A_b$ is endogenous, depending on the price ϕ_b. Despite this, one can check $\partial z_m/\partial \iota < 0$, $\partial z_b/\partial \iota > 0$ as well as the impact on the q's and financial variables.[15] One can also check $\partial z_m/\partial A_b < 0$, $\partial z_b/\partial A_b > 0$, etc. Since the results are qualitatively the same as those above, we return below to short-term bonds.

First, however, to draw out a few additional implications using long- and short-term bonds, it is useful to depict the Euler equations in (z_m, z_b) space as the EM and EB curves shown in Figure 3. Both are downward sloping,

$$\begin{aligned}
\frac{\partial z_b}{\partial z_m}_{|EM} &= -\frac{\alpha_m L'(z_m) + \alpha_2 L'(z_m + z_b)}{\alpha_2 L'(z_m + z_b)} < 0 \\
\frac{\partial z_b}{\partial z_m}_{|EB} &= \frac{\alpha_2 L'(z_m + z_b)}{r - \gamma(1 + r) B/z_b - \alpha_b L'(z_b) - \alpha_2 L'(z_m + z_b)} < 0,
\end{aligned}$$

but one can show EB has a greater slope and they cross just once. An increase in i shifts EM to the southwest but does not affect EB, while an increase in B shifts EB to the northeast but does not affect EM. Hence either an an increase in i or an increase in A_b lowers z_m and raises z_b, as shown in the top two panels. For comparison, the bottom two panels show the analogous results in the model with short-term (one-period) bonds, where z_b is an exogenous policy instrument.

[14] Geromichalos et al. (2013), Rocheteau and Rodriguez-Lopez (2013) and Williamson (2014a) have assets that 'die' each period with some probability δ, where $\delta = 1$ is like our short- and $\delta = 0$ our long-term assets. As in those papers, this has implications for the term premium.

[15] In particular, again $\partial \rho/\partial \iota$ is ambiguous due to the competing Fisher and Mudell effects.

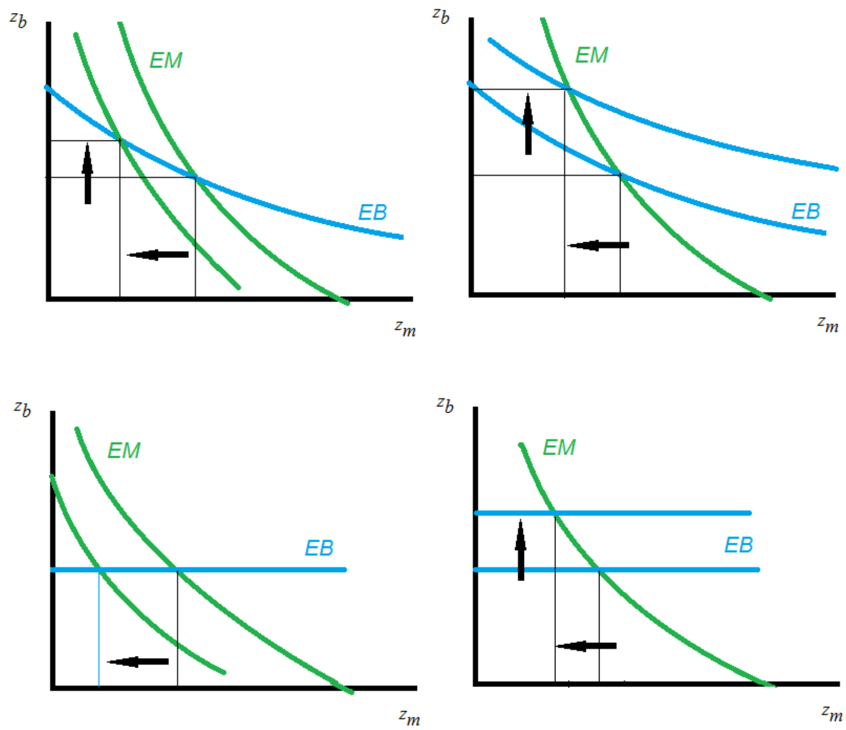

Figure 3: Effects of increases in ι and A_b with long- and short-term bonds

As in Section 3.1, an OMO puts cash into the hands of the public by buying bonds can be decomposed into two effects. The direct effect of increasing A_m on real variables is nil since ϕ_m adjusts to leave z_m constant. But the fall in A_b stimulates demand for z_m, to make up for the contraction in bond liquidity, so on net ϕ_m goes down by less than A_m goes up. In the lower right panel of Figure 3, with short-run bonds, EB shifts down after an OMO increasing A_m and decreasing A_b (the arrows depict A_b increasing). Since z_m increases, prices do not respond one-for-one to the increase in the money supply. This looks like sticky nominal prices. The upper right panel, with long-term bonds, is similar but there is an additional *multiplier* effect. After the OMO shifts EB down there is a rise in z_m for a given z_b, but now the rise in z_m further reduces z_b, etc. This looks like very sticky prices. But in neither case is there any *genuine* nominal rigidity, in the sense that an increase in A_m accomplished by a lump sum transfers, say, instead of OMO's necessarily entails a one-for-one change in ϕ_m. Now, it may be nontrivial in general to distill from data the impact of these different kinds of monetary injections, but at least in theory the results are clear.[16]

3.3 A Liquidity Trap

A recurring theme below is that there can emerge outcomes resembling liquidity traps, where OMO's do not affect ρ, or the allocation, which involves inefficiently low q's. While Figure 2 shows changes in A_b are neutral when bonds are abundant, that is because agents can get satiated in bond liquidity. Now, for something completely different, consider incorporating heterogeneous buyers, where type-j have probabilities α_m^j, α_b^j and α_2^j of type-m, type-b and type-2 meetings.[17]

Let μ_j be the fraction of type-j buyers, and suppose there is a type j with $\alpha_m^j = \alpha_b^j = 0 < \alpha_2^j$. If they choose $\hat{z}_m^j > 0$ and $\hat{z}_b^j > 0$, where superscripts here indicate type, their Euler equations are

$$1 + \pi = \beta \left[1 + \alpha_2^j \chi_m \lambda(q_2^j)\right] \tag{12}$$

$$\phi_b = \beta \left[1 + \alpha_2^j \chi_b \lambda(q_2^j)\right] \tag{13}$$

As a special case of (10), special because for this type $\alpha_m^j = \alpha_b^j = 0$, we get

$$\rho = \underline{\rho} = \frac{(\chi_m - \chi_b)\,\iota}{\chi_m + \iota\chi_b},$$

which pins down the nominal T-bill rate as a function of ι and the χ's. Intuitively,

[16] These results are different from recent New Monetarist papes where nominal prices are endogenously sticky in the cross section of sellers because of search frictions (Head et al. 2012; Liu et al. 2014), but the general message is similar: it is hard to tell from data if prices are *genuinely* sticky, in the sense used in the text, and the appearance of stickiness does not logically imply Keynesian models or policy implications are correct. Of course this message can also be found in Old Monetarist papers, including famously Friedman (1968) and Lucas (1972).

[17] We explain the role of heterogeneity below. Also, types can be permanent, or random each period if they are revealed before the CM closes so agents can tailor portfolios appropriately. This is similar to Williamson (2012), but since his types are realized after the CM closes, they use banks to rebalance their portfolios (e.g., buyers more likely to have type-m meetings take out more cash). While integrating banking is interesting, it is not needed for the message here.

for type-j, bonds and money are perfect substitutes because one unit of $\hat{z}_m$ in the DM always gets them the same as χ_b/χ_m units of $\hat{z}_b$. Hence, if $\hat{z}_m^j > 0$ and $\hat{z}_b^j > 0$, the assets must have the same return after adjusting for pledgeability.

Consider two types: type-m buyers have $\alpha_b = \alpha_2 = 0 < \alpha_m$; type-2 buyers have $\alpha_m = \alpha_b = 0 < \alpha_2$. Type-$m$ hold $\hat{z}_m^m > 0$, and we impose $\hat{z}_b^m = 0$ wlog because, just like sellers, they are indifferent to holding bonds if they are priced fundamentally and strictly prefer not to if there is a liquidity premium. For these buyers, $\iota = \alpha_m \chi_m L(\chi_m z_m^m)$ determines z_m and $q_m = v^{-1}(\chi_m z_m^m)$. Type-2 buyers hold all the bonds, $\hat{z}_b^2 = A_b/\mu_2 > 0$, and maybe some cash, $z_m^2 \geq 0$. There are three possibilities. If bonds are plentiful, $A_b \geq A_b^*$ where $\chi_b A_b^*/\mu_2 = v(q^*)$, then $\hat{z}_m^2 = 0$ and $q_2 = q^*$. If $A_b < A_b^*$, there are two subcases. One has $\hat{z}_m^2 = 0$ even though $q_2 < q^*$, and occurs if $A_b \geq \bar{A}_b$ where $\iota = \alpha_2 \chi_m L\left(\chi_b \bar{A}_b/\mu_b\right)$. In this subcase type-2 cannot get q^*, but q_2 is big enough that they choose not to bear the cost ι of topping up liquidity using cash. The other subcase has $\hat{z}_m^2 > 0$, occurs if $A_b < \bar{A}_b$, and implies $\iota = \alpha_2 \chi_m L(\chi_m z_m^2)$. In this subcase total liquidity for type-2 is independent of A_b, because at the margin *it's money that matters.*

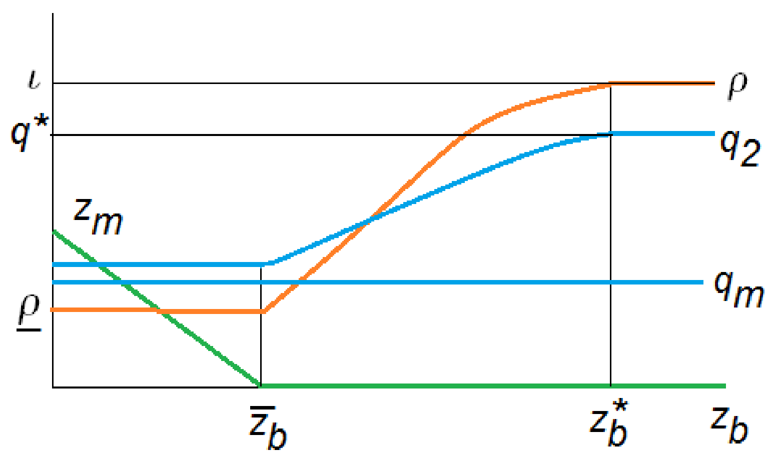

Figure 4: Effects of z_b, with a liquidity trap in $(0, \bar{z}_b)$

As shown in Figure 4, $\hat{z}_m^2 > 0$ occurs for $A_b \in (0, \bar{A}_b)$. As A_b increases over this interval, there is one-for-one crowding out of z_m^2 until it hits 0 at $\bar{A}_b$.

over the range $(0, \bar{A}_b)$ we are in a liquidity trap, where OMO's are ineffectual, changes in A_b induce changes in real balances so that total liquidity stays the same, while ρ and the q's are stuck at their lower bounds. In terms of economics, in this regime $\hat{z}_m^2$ and $\hat{z}_b^2$ are perfect substitutes for type-2, and hence must have the same pledgeability-adjusted return. This is related to Wallace's (1981,1983) result for OLG models, although, in our notation, he had $\alpha_m = \alpha_b = 0$ and $\alpha_2 = \chi_m = \chi_b = 1$, meaning that $\hat{z}_m$ and $\hat{z}_b$ are always perfect substitutes. To clarify why we have two types, we want some agents to go through different regimes as A_b increases, from using money and getting $q < q^*$, to not using money but still getting $q < q^*$, to eventually getting $q = q^*$. This is the role of type-2. But we do not want monetary equilibrium to collapse for $A_b \geq A_b^*$. Thus we include type-m, who always need cash, and hence prop up the value of currency. And that is our rudimentary model of a liquidity trap.

4 Endogenous Liquidity

Can α_j and χ_j arise from the environment? While there are different ways to contemplate this, we follow a long tradition (see Lagos et al. 2014, Sec. 11) appealing to *recognizability* – i.e., asymmetric information about asset quality.[18]

4.1 Acceptability

Following Lester et al. (2012) we now assume: 1. some sellers cannot distinguish high- and low-quality versions of certain assets; 2. low-quality assets have 0 value; and 3. they can be produced on the spot for free. Section 4.2 below changes 3, but in any case, to guarantee low-quality assets are worth 0, think of them as fraudulent or counterfeit and assume anyone getting stuck with one in the DM has it authenticated and confiscated in the next CM (a specification borrowed

[18] An alternative approach studies pairwise-efficient trading mechanisms that treat assets asymmetrically, which under some conditions can be socially efficient. See Zhu and Wallace (2009), Nosal and Rocheteau (2013) or Hu and Rocheteau (2013,2014).

from Nosal and Wallace 2007). Then sellers unable to recognize quality must reject an asset outright – if they were to accept it, buyers would simply hand over worthless paper. This is extreme but convenient relative to buyers choosing asset quality before knowing if they will meet an informed or uninformed counterparty (Williamson and Wright 1994; Berentsen and Rocheteau 2004). In those settings generally sellers accept unrecognized assets with some probability; here they reject them outright, which allows us to avoid bargaining under private information even while recognizability drives acceptability.

Since having an exogenous set of agents recognize A_m, A_b or both is no 'deeper' than having a fixed set accepting the assets, let us endogenize this, as follows. Normalize the measure of sellers to $n = 1$, let n_2 be the measure that recognize both assets, and let $n_m = 1 - n_2$ be the measure that recognize only money, assuming here that no one recognizes only bonds. While everyone recognizes currency, for free, sellers must pay an individual-specific cost κ to distinguish a bond's quality, with CDF $F(\kappa)$. Thus, κ is a cost of information, or maybe a technology, like counterfeit-detection devices. As in Lester et al. (2012), set $\chi_j = 1$ and use Kalai bargaining, $v(q) = \theta c(q) + (1-\theta) u(q)$. Then the marginal seller is one with $\kappa = \Delta$, where from the bargaining solution

$$\Delta = \alpha (1-\theta) [u(q_2) - c(q_2) - u(q_m) + c(q_m)] = \frac{\alpha (1-\theta)}{\theta} [u(q_2) - u(q_m) - \hat{z}_b]$$

is the net increase in flow profit from being informed.

Equilibrium for sellers entails $n_2 = F(\Delta)$, with $\Delta = \Delta(z_m)$ because $v(q_m) = z_m$ and $v(q_2) = z_m + z_b$. Thus $n_2 = F \circ \Delta(z_m)$ defines a curve in (n_2, z_m) space called IA, for information acquisition, that slopes down and shifts right with A_b:

$$\begin{aligned} \frac{\partial n_2}{\partial z_m}_{|IA} &= \frac{\alpha (1-\theta) F'(\Delta) [c'(q_m) u'(q_2) - c'(q_2) u'(q_m)]}{v'(q_m) v'(q_2)} < 0 \\ \frac{\partial n_2}{\partial A_b}_{|IA} &= \frac{\alpha (1-\theta) F'(\Delta) [u'(q_2) - c'(q_2)]}{v'(q_2)} > 0. \end{aligned}$$

For buyers, assume they are constrained in all meetings, as in Case 1 above. Then the Euler equation $\iota = \alpha (n - n_2) L(z_m) + \alpha n_2 L(z_m + z_b)$ defines a curve called RB, for real balances, that slopes down and shifts down as A_b increases:

$$\begin{aligned}
\frac{\partial z_m}{\partial n_2}_{|RB} &= \frac{L(z_m)}{n_2 L'(z_m + z_b) + n_m L'(z_m)} < 0 \\
\frac{\partial z_m}{\partial A_b}_{|RB} &= \frac{-n_2 L'(z_m + z_b)}{n_2 L'(z_m + z_b) + n_m L'(z_m)} < 0,
\end{aligned}$$

Also note that increases in i shift RB down but do not affect IA. These observations help sort out the effects of policy below.

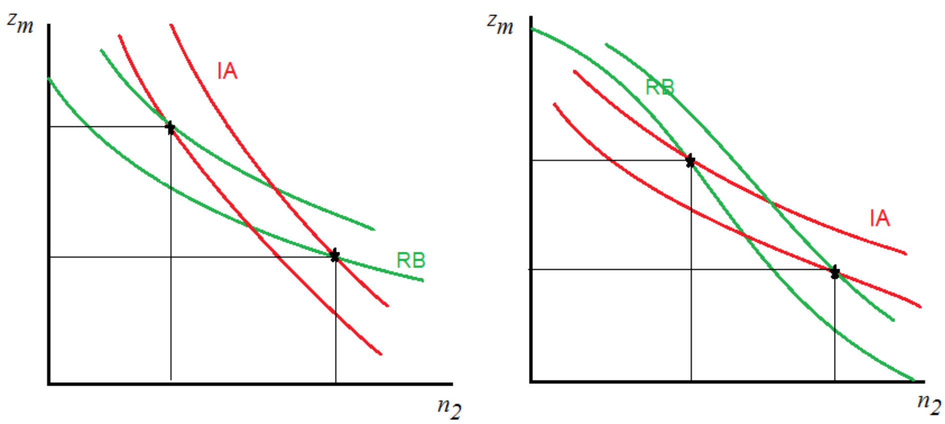

Figure 5: Equilibria with endogenous α's

As shown in Figure 5, RB can cut IA from below or from above. In either case, $n_2 = F \circ \Delta(z_m)$ is decreasing in z_m via IA and z_m is decreasing in n_2 via RB. Hence, equilibrium involves $n_2 = F \circ \Delta \circ z_m (n_2) \equiv \Phi(n_2)$ where $\Phi : [0,1] \to [0,1]$ is increasing. Existence therefore follows by Tarski's fixed-point theorem, even when $F(\cdot)$ is not continuous, as when there is a mass of sellers with the same κ. Since Φ is increasing, *multiplicity* can easily emerge, intuitively, because higher n_2 decreases z_m, which raises the relative profitability of recognizing bonds, which increases the fraction of sellers investing in information n_2. This can also lead to *fragility*, with small changes in parameters causing jumps in (z_m, n_2). Also, while we are assuming κ must be paid every period, it is not hard to change this and make it a one-time fixed cost. Then the model generates *hysteresis*, as is commonly thought to characterize dollarization episodes, where higher domestic inflation leads to more locals learning to use foreign currency, but subsequent disinflations do not lead to reversals, since they tend not to forget.[19]

Using '$x \backsimeq y$' to mean x and y take the same sign, we have

$$\begin{aligned}
\frac{\partial z_m}{\partial \iota} &= \frac{v'(q_m)v'(q_2)}{D_\alpha} \backsimeq D_\alpha \\
\frac{\partial q_m}{\partial \iota} &= \frac{v'(q_2)}{D_\alpha} \backsimeq D_\alpha \\
\frac{\partial q_2}{\partial \iota} &= \frac{v'(q_m)}{D_\alpha} \backsimeq D_\alpha \\
\frac{\partial n_2}{\partial \iota} &= \frac{\Omega\left[c'(q_m)u'(q_2) - c'(q_2)u'(q_m)\right]}{D_\alpha} \backsimeq -D_\alpha
\end{aligned}$$

where $\Omega = \alpha\left(1-\theta\right)F'(\Delta)$ and

$$\begin{aligned}
D_\alpha &= \alpha\Omega\left[\lambda(q_2) - \lambda(q_m)\right]\left[c'(q_m)u'(q_2) - c'(q_2)u'(q_m)\right] \\
&\quad + \alpha n_m \lambda'(q_m)v'(q_2) + \alpha n_2 \lambda'(q_2)v'(q_2).
\end{aligned}$$

Notice $D_\alpha < 0$ iff RB cuts IA from below, in which case higher ι by shifting RB down decreases z_m and increases n_2 along FE. As usual, with multiple equilibria, the results alternate across them. While intuitive results (i.e., ones qualitatively similar to what we had with the α's fixed) follow from $D_\alpha < 0$, it not obvious that this is the only interesting case, and it is easy to find conditions implying there is a unique equilibrium and it has $D_\alpha > 0$. For financial variables, one can check $\partial s/\partial\iota \backsimeq \partial\phi_b/\partial\iota \backsimeq -D_\alpha$, while $\partial\rho/\partial\iota$ is ambiguous, as was already true in Section 3 when the α's were fixed.

[19] Lester et al. (2012) have similar results but emphasize different forces. First, they use long-lived assets as we discussed in Section 3.2. When n_2 is higher these assets are more liquid and hence fetch a higher CM price, which raises the payoff of type-2. We use one-period bonds with fixed CM redemption. When n_2 is higher z_m falls, which reduces type-2 payoffs but reduces type-m payoffs by more, so Δ still increases. Lester et al. (2012) discuss only the CM price effect, which is absent here.

The effects of OMO's are given by

$$\frac{\partial z_m}{\partial A_b} = -\frac{\alpha v'(q_m)\left\{n_2\lambda'(q_2) + \Omega\left[u'(q_2) - c'(q_2)\right]\left[\lambda(q_2) - \lambda(q_m)\right]\right\}}{nD_\alpha} \backsimeq D_\alpha$$
$$\frac{\partial q_m}{\partial A_b} = -\frac{\alpha\left\{n_2\lambda'(q_2) + \Omega\left[u'(q_2) - c'(q_2)\right]\left[\lambda(q_2) - \lambda(q_m)\right]\right\}}{nD_\alpha} \backsimeq D_\alpha$$
$$\frac{\partial q_2}{\partial A_b} = \frac{\alpha\left\{(n - n_2)\lambda'(q_m) + \Omega\left[u'(q_m) - c'(q_m)\right]\left[\lambda(q_2) - \lambda(q_m)\right]\right\}}{nD_\alpha} \backsimeq -D_\alpha$$
$$\frac{\partial n_2}{\partial A_b} = \frac{\alpha\Omega\left\{(n - n_2)\left[u'(q_2) - c'(q_2)\right]\lambda'(q_m) + n\left[u'(q_m) - c'(q_m)\right]\lambda'(q_2)\right\}}{nD_\alpha} \backsimeq -D_\alpha.$$

One can also derive the effects on financial variables, which are somewhat complicated, although we can say $\partial s/\partial A_b \backsimeq \partial\phi_b/\partial A_b \backsimeq -\partial\rho/\partial A_b$. In terms of Figure 5, if $D_\alpha < 0$, as in the left panel, an OMO that retires bonds shifts RB up and FE left, thus increasing q_m and decreasing q_2. This was true with fixed α's, too, but here the effects are multiplied: lower A_b initially raises z_m and q_m, then n_2 falls, which leads to a further rise in z_m, which leads to a further fall in n_2, etc.

Then again, if $D_\alpha > 0$, the results are just the opposite. Is it problematic that our predictions depend on whether RB cuts IA from above or below? We think not. This is to be expected in models with multiple equilibria, and multiplicity arises naturally in situations characterized by coordination and complementarities, like the choice of payment instruments. Since our understanding of monetary policy is not enhanced by ignoring the issue, we are content with reporting results contingent on D_α.[20]

4.2 Pledgeability

The next step is to ask if the χ's can emerge endogenously. Following Rocheteau (2011) and Li et al. (2012), suppose agents can produce low-quality assets at costs proportional to their values: the real cost of producing a unit of counterfeit currency is $\tilde{\gamma}_m\phi_m$; and the cost of producing a unit of fake bonds is $\tilde{\gamma}_b$. Different

[20] While we focused here on $q_m < q_2 < q^*$, as in Section 3 one can also consider $q_2 = q^*$ (we leave this as an exercise) and try to constuct a liquidity trap (we return to that below).

from Section 4.1, all sellers are uninformed, and the decision to produce fraudulent assets is made in the CM before visiting the DM, but still fraudulent assets are confiscated in the next CM.[21]

As in standard signaling models, here we use bargaining with $\theta = 1$, and focus for now on $\alpha_2 > 0 = \alpha_m = \alpha_b$ (but see below). Thus, there is only one kind of meeting, but we need to distinguish payments in the meeting made in money and bonds, say d_m and d_b, and not just the sum as in Section 3. This leads to an incentive condition for d_m,

$$(\phi_{m,-1} - \beta\phi_m)\, a_m + \beta\alpha_2\phi_m d_m \leq \tilde{\gamma}_m\phi_m a_m, \tag{14}$$

as shown in Li et al. (2012) (their Appendix B allows fixed and proportional costs of fraud; we ignore the former mainly to reduce notation).

The RHS is the cost of counterfeiting a_m units of currency; the LHS is the cost of acquiring a_m genuine units, $(\phi_{m,-1} - \beta\phi_m)a_m$, plus the cost of giving up d_m with probability α_2. Sellers rationally believe that buyers would not pay with fraudulent assets if (14) holds – who would spend \$20 to spend \$10 worth of counterfeits, or pass a \$10 bad check? One can also interpret $d_m < a_m$ as over-collateralization, or *asset retention*, as a signal of quality.

There is a similar incentive condition for d_b

$$(\phi_{b,-1} - \beta)\, a_b + \beta\alpha_2 d_b \leq \tilde{\gamma}_b a_b. \tag{15}$$

Buyers now have multiple constraints in the DM: bargaining implies $c(q_2) = \phi_m d_m + d_b$; feasibility implies $\phi_m d_m \leq z_m$ and $d_b \leq z_b$; and (14)-(15) imply

[21] As discussed above, instead of information frictions, one can try to justify $\chi_j < 1$ by saying borrowers can abscond with a fraction of the collateral. As pointed out to us by Ricardo Cavalcanti, however, making this rigorous requires additional assumptions about the institutions monitoring compliance and seizing assets from defaulters. This is different from assumptions made to render low-quality assets worthless – that can be guaranteed by conditions on the physical environment, like having counterfeits fully depreciate in a period. Again, this not meant to be a 'deep' insight; we are merely acknowledging that there are subtleties involved.

$d_j \leq \chi_j z_j$ with

$$\chi_m = \frac{\gamma_m - \iota}{\alpha_2} \text{ and } \chi_b = \frac{\gamma_b - s}{\alpha_2}, \tag{16}$$

where $\gamma_j = \tilde{\gamma}_j/\beta$. Notice $\partial\chi_j/\partial\alpha_2 < 0$, because fraud is more tempting when there are more opportunities to pass bad assets. Also, $\partial\chi_m/\partial\iota < 0$ and $\partial\chi_b/\partial s < 0$, so pledgeability like acceptability is not invariant to economic conditions.

Again there are different cases, depending on which incentive conditions bind. Starting with the regime where they both bind, $\chi_j \in (0, 1)$, we reduce equilibrium conditions (8) and (9) to

$$\iota = (\gamma_m - \iota)\,\lambda(q_2) \text{ and } s = (\gamma_b - s)\,\lambda(q_2), \tag{17}$$

using (16). The first equality yields q_2. Then the second yields $s = \iota\gamma_b/\gamma_m$, which further implies $\chi_b = \gamma_b\,(\gamma_m - \iota)\,/\alpha_2\gamma_m$, and that tells us this case, with $\chi_j \in (0, 1)$, obtains iff $\gamma_m > \iota$, $\gamma_m < \iota + \alpha_2$ and $\gamma_b < \alpha_2\gamma_m/\,(\gamma_m - \iota)$. Notice

$$\frac{\partial q_2}{\partial \iota} = \frac{1 + \lambda\,(q_2)}{(\gamma_m - \iota)\,\lambda'\,(q_2)} < 0,$$

which is different from what we found with the χ's fixed because now they respond to policy. From (10), the nominal rate ρ satisfies $1 + \rho = \chi_m/\chi_b = \gamma_m/\gamma_b$. Thus, $\rho < 0$ iff bonds are more pledgeable than cash, which now translates into cash being easier to counterfeit. Either way, OMO's affect neither q_2 nor ρ because, once again, z_m and z_b are perfect substitutes after adjusting for pledgeability.[22]

Moving to the next, and perhaps most realistic, regime, $\chi_m = 1$ and $\chi_b \in [0, 1)$, we reduce the equilibrium conditions to

$$\iota = \alpha_2\lambda(q_2) \text{ and } s = (\gamma_b - s)\,\lambda(q_2). \tag{18}$$

Now $\partial q_2/\partial\iota$ is the same as it would be if χ_m were fixed, because it is fixed, at

[22] As always, we assume money is valued, which here requires $\iota < (\gamma_m - \iota)\,\lambda(\tilde{q})$ with $\tilde{q}$ defined by $c(\tilde{q}) = \gamma_b z_b(\gamma_m - \iota)/\alpha_2\gamma_m$.

$\chi_m = 1$. Also, $s = \iota\gamma_b / (\iota + \alpha_2)$, so the regime $\chi_m = 1$ and $\chi_b \in [0, 1)$ requires $\gamma_m > \iota + \alpha_2$ and $\gamma_b < \iota + \alpha_2$. Moreover, $\rho = \iota(\iota + \alpha_2 - \gamma_b) / (\iota + \alpha_2 + \iota\gamma_b)$. And it is still true that OMO's affect neither q_2 nor ρ, for the same reason.[23]

One can similarly consider the regime $\chi_b = 1$ and $\chi_m \in [0, 1)$, or the regime $\chi_m = \chi_b = 1$. Also, while there is no monetary equilibrium if $\gamma_m \leq \iota$, there are nonmonetary equilibria where A_b is the only means of payment. Figure 6 displays the regions of (γ_b, γ_m) space where the different regimes constitute equilibria, illustrating how pledgeability depends on frictions and policy. In all of these equilibria, OMO's are neutral because bonds and currency are perfect substitutes after adjusting for the now-endogenous pledgeability factors.

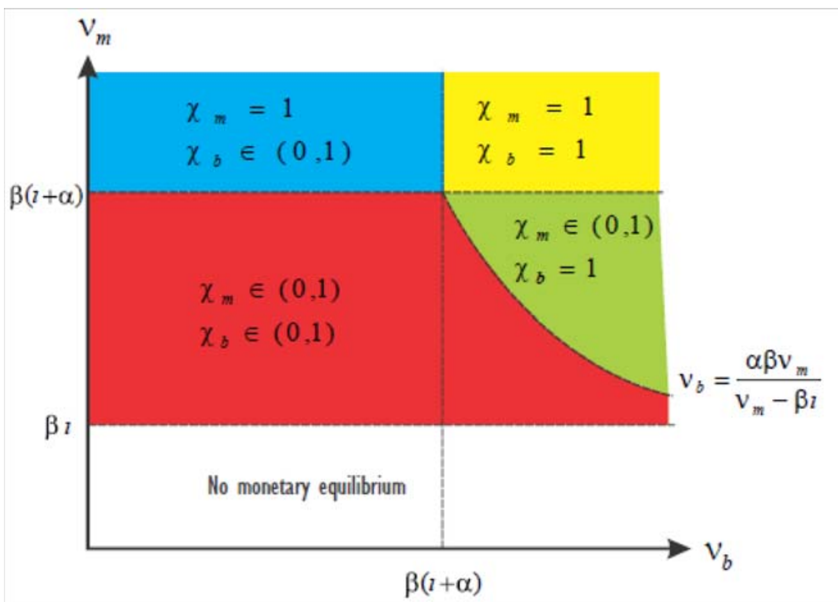

Figure 6: Different regimes with endogeneous χ's

Based on versions of the model presented above, we also know how to change the environment so that OMO's are not neutral: make assumptions so that z_m and z_b are not perfect substitutes. Consider the regime $\chi_m = 1$, $\chi_b \in (0, 1)$, and now let $\alpha_m > 0$, which means that in some meetings bonds are not accepted. The equilibrium conditions are (8)-(9), with $\chi_b = (\gamma_b - s) / \alpha_2$, which implies

[23] Again we assume money is valued, which requires $\iota < \alpha_2 \lambda(\tilde{q})$ with $c(\tilde{q}) = \gamma_b z_b / (\iota + \alpha_2)$.

$s = \gamma_b \lambda(q_2) / [1 + \lambda(q_2)]$ and $\chi_b = \gamma_b / \alpha_2 [1 + \lambda(q_2)]$, and so $\chi_b < 1$ requires $\gamma_b < \alpha_2 [1 + \lambda(q_2)]$. Using the bargaining solution, we collapse the system to

$$\iota = \alpha_m L(z_m) + \alpha_2 L(z_m + \chi_b z_b)$$
$$\gamma_b = \alpha_2 \chi_b [1 + L(z_m + \chi_b z_b)],$$

defining two curves in (χ_b, z_m) space labeled RB and IC in Figure 7.

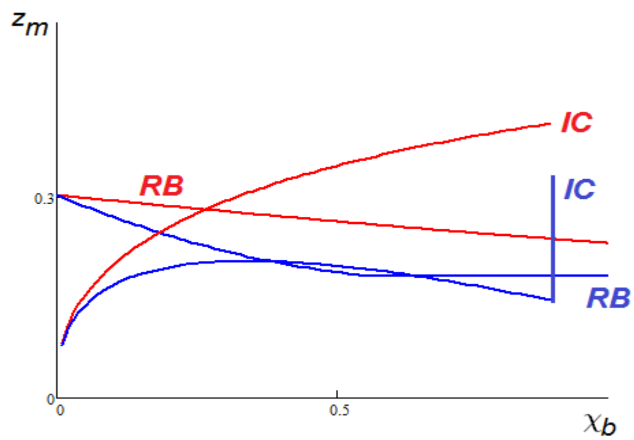

Figure 7: Uniqueness (red) or multipliity (blue) with endogenous χ_b

The RB curve is slopes down,

$$\frac{\partial z_m}{\partial \chi_b}_{|RB} = \frac{-\alpha_2 z_b L'(z_m + \chi_b z_b)}{\alpha_m L'(z_m) + \alpha_2 L'(z_m + \chi_b z_b)} < 0,$$

and shifts down with higher ι or z_b. The IC curve can be nonmonotone,

$$\frac{\partial z_m}{\partial \chi_b}_{|IC} \backsimeq \Theta \equiv 1 + L(z_m + \chi_b z_b) + \chi_b z_b L'(z_m + \chi_b z_b),$$

and shifts down with higher γ_b or A_b. An example with multiple equilibria is shown in Figure 7 in blue, and one with uniqueness in red.[24] As regards multiplicity, intuitively, if agents believe χ_b is low then q_2 is low, so $\lambda(q_2)$ and s are big,

[24] Here $u(q) = \left[(q+\varepsilon)^{1-a} - \varepsilon^{1-a}\right] / (1-a)$ and $c(q) = q^{1+\delta} / (1+\delta)$ with $a = 4$, $\delta = 1$, $\varepsilon = 0.01$, $\beta = 0.9$, $\alpha_m = 0.1$, $\alpha_2 = 0.5$, $\gamma_b = 0.44$, $i = 1.5$ and $z_b = 0.1$ (red) or $z_b = 0.4$ (blue).

and then the incentive for fraud is high, consistent with low χ_b. Note the blue curves generate two equilibria with $q_2 < q^*$, plus one with $q_2 = q^*$ on the vertical segment of IC. Thus, a change in A_b can change the structure of the equilibrium set by shifting RB, so again liquidity is fragile.

As in Section 4.1, what matters for many results is the relative slopes of RB and IC, which depends on

$$D_\chi = \left[1 + L\left(z_m + \chi_b z_b\right)\right]\left[\alpha_m L'\left(z_m\right) + \alpha_2 L'\left(z_m + \chi_b z_b\right)\right] + \alpha_m \chi_b z_b L'\left(z_m + \chi_b z_b\right) L'\left(z_m\right).$$

As defined above, Θ gives the slope of IC, and $\Theta > 0$ implies $D_\chi < 0$. Hence there are three relevant configurations: (i) $\Theta > 0$ implies IC is upward sloping and cuts RB from below, as at point a in the left panel of Figure 8; $\Theta < 0$ implies IC is downward sloping and either (ii) cuts RB from below, as at point e in the right panel, or cuts it from above, as at c or g. An increase in ι shifts RB down. This can perversely make $\partial z_m/\partial \iota = \Theta/D_\chi > 0$ when $\Theta < 0$, as in the move from e to f, or can more naturally make $\partial z_m/\partial \iota < 0$, as in all other cases. It can also increase or decrease χ_b, since $\partial \chi_b/\partial \iota \simeq D_\chi$.

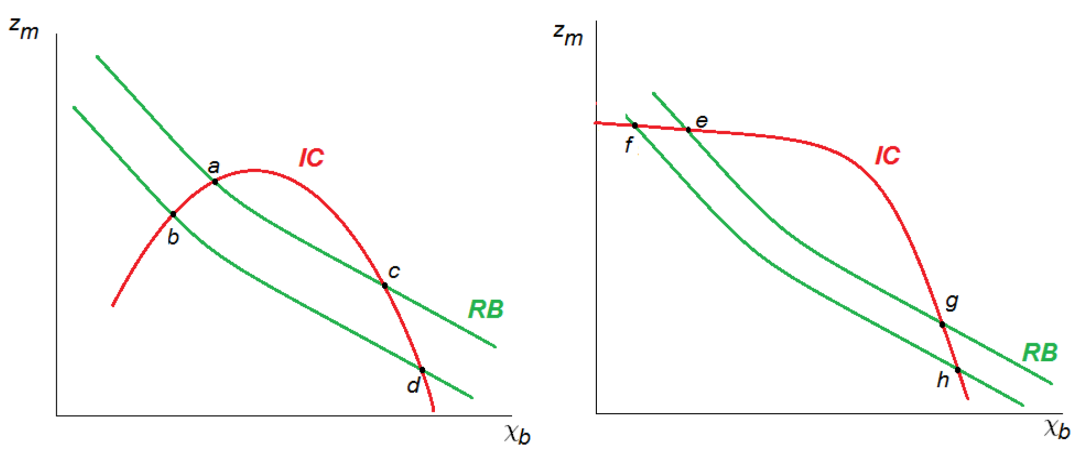

Figure 8: Different configurations with endogenous χ's

This makes clear how $\partial z_m/\partial \iota \simeq \partial q_m/\partial \iota$ depends on the configuration of RB

and IC. Other effects of ι include

$$
\begin{aligned}
\frac{\partial q_2}{\partial \iota} &= \frac{\alpha_2 \left[1+\lambda(q_2)\right] \upsilon'(q_m)}{D} \simeq D \\
\frac{\partial \chi_b}{\partial \iota} &= -\frac{\alpha_2 \chi_b \upsilon'(q_m) \lambda'(q_2)}{D} \simeq D \\
\frac{\partial s}{\partial \iota} &= \frac{\alpha_2 \gamma_b \upsilon'(q_m) \lambda'(q_2)/\left[1+\lambda(q_2)\right]}{D} \simeq -D,
\end{aligned}
$$

with $D = \alpha_m \alpha_2 \chi_b z_b \lambda'(q_m)\lambda'(q_2) + \alpha_2 \left[1+\lambda(q_2)\right] \left[\alpha_m \lambda'(q_m)\upsilon'(q_2) + \alpha_2 \lambda'(q_2)\upsilon'(q_m)\right]$, which again depend on the configuration. One can also check $\partial \phi_b / \partial \iota \simeq -D$. However, $\partial \rho / \partial \iota$ is ambiguous, as was already true in Section 3 when χ's were fixed.

OMO's have the following effects:

$$
\begin{aligned}
\frac{\partial z_m}{\partial A_b} &= -\frac{\alpha_2^2 \chi_b \left[1+\lambda(q_2)\right] \upsilon'(q_m)\lambda'(q_2)}{D} \simeq D \\
\frac{\partial q_m}{\partial A_b} &= -\frac{\alpha_2^2 \chi_b \left[1+\lambda(q_2)\right] \lambda'(q_2)}{D} \simeq D \\
\frac{\partial q_2}{\partial z A_b} &= \frac{\alpha_m \alpha_2 \chi_b \left[1+\lambda(q_2)\right] \lambda'(q_m)}{D} \simeq -D \\
\frac{\partial \chi_b}{\partial A_b} &= -\frac{\alpha_m \alpha_2 \chi_b \lambda'(q_m)\lambda'(q_2)}{D} \simeq -D.
\end{aligned}
$$

Changing A_b moves z_m and q_m one way, and q_b and χ_c the other. Similarly,

$$
\begin{aligned}
\frac{\partial s}{\partial A_b} &= \frac{\alpha_m \alpha_2 \chi_b \gamma_b \lambda'(q_m)\lambda'(q_2)/\left[1+\lambda(q_2)\right]}{D} \simeq D \\
\frac{\partial \phi_b}{\partial A_b} &= \beta \frac{\alpha_m \alpha_2 \chi_b \gamma_b \lambda'(q_m)\lambda'(q_2)/\left[1+\lambda(q_2)\right]}{D} \simeq D \\
\frac{\partial \rho}{\partial A_b} &= -\frac{1+\rho}{1+s} \frac{\alpha_m \alpha_2 \chi_b \gamma_b \lambda'(q_m)\lambda'(q_2)/\left[1+\lambda(q_2)\right]}{D} \simeq -D.
\end{aligned}
$$

As in Section 4.1, theory delivers sharp predictions conditional on D. A bigger point is that it is possible to study the effects of OMO's rigorously, based on liquidity considerations, with pledgeability and acceptability endogenous.

5 Directed Matching

Instead of agents meeting at random, suppose that buyers can *direct* their search, based on the payment instruments that sellers accept and, in one version of the model analyzed below, on the terms of trade. One reason this is relevant is that directed search generates endogenous market segmentation, and while there is already a literature on monetary policy with segmented markets, mentioned in the Introduction, our theory is different and, we think, natural. To reduce notation, set $\chi_j = 1$, and while there can in principle be three types of sellers consider only two: a measure n_m accept only money while a measure $n_2 = n - n_m$ accept money and bonds. For now we normalize $n = 1$ and fix n_j.

While a buyer can choose to search for a given type of seller, whether he finds one is still random. Define submarket j by a set of type-j sellers, with measure n_j, and a set of buyers that choose to look for them with measure μ_j. Let *SM* denote the submarket where only money is accepted and *S2* the one where both assets are accepted. The total measure of buyers is $\mu_m + \mu_2 = \mu$, where μ is assumed to be not too large, so that all buyers want to participate (see below). The matching technology is the same in each submarket and satisfies constant returns, so what matters is not the size of a submarket but tightness, as captured by the seller-buyer ratio, $\sigma_j = n_j/\mu_j$. This means the probability a buyer meets a seller is $\alpha(\sigma_j)$, and the probability a seller meets a buyer is $\alpha(\sigma_j)/\sigma_j$, with the former increasing and the latter decreasing in σ_j.

5.1 Bargaining

We start by assuming the terms of trade are negotiated using Kalai bargaining, $v(q) = \theta c(q) + (1-\theta)u(q)$. As in Section 3.3, buyers going to *SM* bring only cash, $\hat{z}_m^m > 0$ and $\hat{z}_b^m = 0$, while those going to *S2* bring bonds plus maybe cash, $\hat{z}_b^2 = A_b/\mu_2 > 0$ and $\hat{z}_m^2 \geq 0$, where superscripts indicate submarkets. Bargaining implies $\hat{z}_m^m = v(q_m)$ and $\hat{z}_b^2 + \hat{z}_m^2 \geq v(q_2)$, where the latter holds with equality iff

$q_2 < q^*$ and $s > 0$ (when bonds are scarce they command a premium). When $q_2 < q^*$, we may have $\hat{z}_m^2 = 0$ or $\hat{z}_m^2 > 0$, with $\iota > s$ in the former case and $\iota = s$ in the latter (buyers hold both assets only if they are perfect substitutes).

As buyers, by choice, meet only one type of seller, the Euler equations are

$$\iota = \alpha(\sigma_m)\left[\lambda(q_m) - 1\right] \text{ and } s = \alpha(\sigma_2)\left[\lambda(q_2) - 1\right]. \tag{19}$$

If *SM* and *S2* are both open, with directed search, buyers must be indifferent between them:

$$\alpha(\sigma_m)\left[u(q_m) - v(q_m)\right] - \iota z_m = \alpha(\sigma_2)\left[u(q_2) - v(q_2)\right] - s z_b. \tag{20}$$

This uses the fact that the payoff in *S2* is equal to the payoff if an individual buyer only holds bonds. And since the total measure of buyers is μ,

$$\frac{n_m}{\sigma_m} + \frac{n_2}{\sigma_2} = \mu. \tag{21}$$

A monetary equilibrium is a list (q_j, σ_j, s) solving (19)-(21).

Again there are three regimes: bonds are plentiful and $q_2 = q^*$; bonds are less plentiful and $q_2 < q^*$ but still type-2 carry no cash; or bonds are so scarce type-2 carry cash plus bonds. Suppose first type-2 carry cash, which means $\rho = 0$ and $s = \iota$. Then (20)-(21) imply $\alpha(\sigma_2) = \alpha(\sigma_m)$ and $\sigma_m = \sigma_2 = \sigma = 1/\mu$. From (19) $q_m = q_2 = q$, where $\iota = \alpha(\sigma)\left[\lambda(q) - 1\right]$. As z_b and z_m are perfect substitutes in this regime, the two submarkets are essentially the same, and increases in A_b merely crowd out real money balances in *S2*. This is consistent with equilibrium when $A_b \leq \underline{A}_b = n_2 \mu v(q)$, which means intuitively that bonds are scarce enough that buyers in *S2* compete with those in *SM* for cash.[25]

[25] Again it is maintained that money is valued, which now requires $\iota < \alpha(\sigma_m)\theta/(1-\theta)$.

Consider next regime $\hat{z}_m^2 = 0$, with $s = 0$ and $q_2 = q^* > q_m$. From (20)

$$\alpha(\sigma_2) = \frac{\max_q \{-\iota v(q) + \alpha(\sigma_m)[u(q) - v(q)]\}}{u(q^*) - v(q^*)}, \tag{22}$$

so $\sigma_2 < \sigma_m$ given $\iota > 0$, which say buyers trade with a lower probability in *S2*. It is easy to check there is a unique (σ_m, σ_2) solving (21)-(22), and this regime is consistent with equilibrium iff $A_b \geq A_b^* \equiv n_2 v(q^*)/\sigma_2$.

Finally, consider $z_m^2 = 0$, $0 < s < \iota$ and $q_2 < q^*$. Buyer indifference means

$$\begin{aligned} -\iota z_m + \alpha(\sigma_m)[u(q_m) - v(q_m)] &= -s z_b + \alpha(\sigma_2)[u(q_2) - v(q_2)] \\ &> -\iota z_m + \alpha(\sigma_2)[u(q_m) - v(q_m)], \end{aligned} \tag{23}$$

where the inequality follows from the fact that bonds are less costly to hold then cash and $q_m < q_2$ from (19). Consequently, $\alpha(\sigma_m) > \alpha(\sigma_2)$, and the probability of trade is higher in *SM*. One can again show existence and uniqueness in this regime, and that it is consistent with equilibrium iff $A_b \in (\underline{A}_b, A_b^*)$.

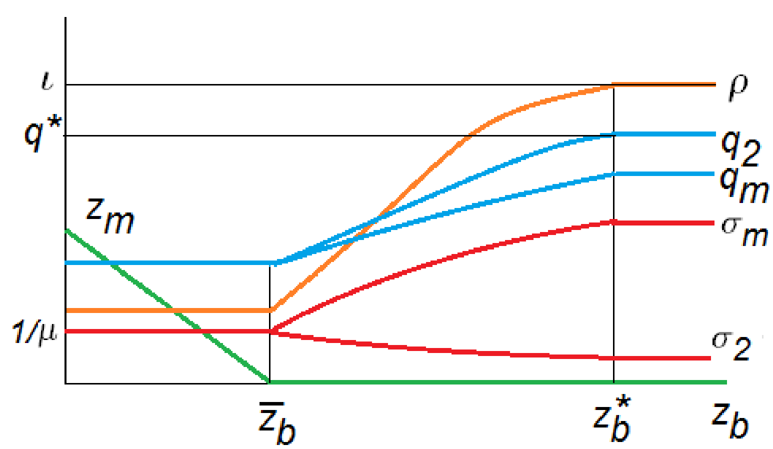

Figure 9: Effects of z_b with directed search

Figure 9 shows the outcome, which should be compared to Figure 4 with random search. As in that situation, where buyers were exogenously partitioned into type-m and type-2, OMO's are neutral when A_b is high or low but not in the intermediate range. Also, in that range, ρ is again increasing in A_b, but now

σ_2 and σ_m depend on A_b because it affects the measures of buyers in *S2* and *SM*. Moreover, now q_m varies with A_b. So while some insights are similar, others change when we segment the market endogenously based on payment method and market tightness considerations. In particular, when A_b increases, buyers in *SM* are made better off on both the intensive and extensive margin – better terms plus a higher probability of trade – even though bonds are not even used in *SM*. Therefore we can have q_b and q_m both increasing in A_b. Comparing this to the baseline model shown in Figure 2, and not the liquidity trap in Figure 4, with random search q_b increases but q_m decreases with A_b. The intuition behind Figure 2 is that higher A_b tends to raise q_2, and this lowers demand for z_m because it is a substitute for z_b in type-2 meetings, and that lowers q_m. With directed search, in the range where A_b matters, $z_m^2 = 0$ and $z_b^m = 0$, so there is no substitution between z_m and z_b.

5.2 Posting

Now assume that search can be directed along two dimensions: the means of payment accepted by sellers; and the posted terms of trade. While there are different ways to do directed search with posting, one approach appeals to third parties called *market makers* that set up submarkets in the DM to attract buyers and sellers, who then meet bilaterally according to a standard matching technology. In the CM market makers post $\left(q_j, \hat{z}_m^j, \hat{z}_b^j, \sigma_j\right)$ for the next DM, where traders commit to swapping q_j for a portfolio $\left(\hat{z}_m^j, \hat{z}_b^j\right)$ if they meet, and meetings are determined by σ_j.[26]

In *SM*, market makers design $(q_m, \hat{z}_m, \sigma_m)$ to maximize buyers' surplus subject

[26] As discussed in Moen (1997) or Mortensen and Wright (2002), the motive of market makers is to charge entrance fees, although in equilibrium competition drives these fees to 0. Usually it does not matter if sellers or buyers, instead of market makers, post the terms; Faig and Huangfu (2007) provide an exception, but that can be finessed as in Rocheteau and Wright (2005). Other papers with money, directed search and posting include Lagos and Rocheteau (2005), Faig and Jerez (2006), Huangfu (2009), Dong (2011) and Dutu et al. (2011). Although they all use similar environments, those papers do not consider two assets or analyze OMO's.

to sellers getting a surplus Π_m, or vice-versa, with the surpluses in equilibrium dictated by the market. Assuming Π_m is not too big, since otherwise the market does not open, the problem reduces to

$$U^b(\iota, \Pi_m) = \max_{q,\hat{z},\sigma} \left\{ \alpha(\sigma) \left[u(q) - \hat{z} \right] - \iota \hat{z} \right\} \text{ st } \frac{\alpha(\sigma)}{\sigma} \left[\hat{z} - c(q) \right] = \Pi_m. \tag{24}$$

The expected payoff of the buyer, $U^b(\iota, \Pi_m)$, is decreasing in both ι and Π_m. Generically (24) has a unique solution (for some values of Π_m there may be multiple solutions, but they are payoff equivalent). Hence, we proceed assuming all submarkets are the same, or, by constant returns, there is just one. Then use the constraint to eliminate $\hat{z}_m$ and take FOC's wrt q_m and σ_m to get

$$\frac{u'(q)}{c'(q)} - 1 = \frac{\iota}{\alpha(\sigma)} \tag{25}$$

$$\alpha'(\sigma) \left[u(q) - c(q) \right] = \Pi_m \left\{ 1 + \frac{\iota \left[1 - \varepsilon(\sigma) \right]}{\alpha(\sigma)} \right\}, \tag{26}$$

where $\varepsilon(\sigma) \equiv \sigma \alpha'(\sigma) / \alpha(\sigma) \in (0, 1)$ is the elasticity of matching.

We can similarly analyze *S2*, with ι replaced by s, Π_m replaced by Π_2, and $U^b(\iota, \Pi_m)$ replaced by $U^b(s, \Pi_2)$. Then a monetary equilibrium is a list $\left(q_j, \hat{z}_m^j, \hat{z}_b^j, \sigma_j, s_j, \Pi_j \right)$ such that $\left(q_j, \hat{z}_m^j, \hat{z}_b^j, \sigma_j \right)$ solves the market-maker problem given Π_j and s_j, $s_m = \iota$ and $s_2 = s$ are determined by market clearing, (21) holds, and $U^b(i, \Pi_m) = U^b(s, \Pi_2)$. As is standard in these kinds of models, there are technical complications due to the fact that the objective function in (24) is not necessarily well behaved (even when α and u are concave their product may not be). Hence it is hard to prove monotonicity of the solution wrt exogenous variables, or even continuity, but in examples the outcomes look qualitatively just like Figure 9.[27] In particular, with directed search and posting, as with directed search and bargaining, when A_b is low *SM* and *S2* have the same (q, σ) and the

[27] Simple example are provided by the matching function $BS/(B+S)$, where B and S are the measures of buyers and sellers, which implies $\varepsilon(\sigma) = 1/(1+\sigma)$. It is easy to verify the outcome looks like Figure 9 using $u(q) = 2\sqrt{q}$, $c(q) = q$, $n_m = 0.3$, $n_2 = 0.7$ and $\iota = 0.1$.

economy is stuck in a trap where OMO's do not move q or ρ, which are at their lower bounds, because again at the margin *it's money that matters*.

While the Appendix provides details for the general case, here we use a special matching function that allows a sharp analytical characterization, $\alpha(\sigma) = \min\{1, \sigma\}$. In fact, this can be interpreted as eliminating search frictions by having everyone on the short side of the market match with probability 1 while the long side is rationed (Nosal and Rocheteau 2011 study this version in a one-asset model). Also, suppose that $\mu > n_m + n_2$, which implies that not all buyers participate in the DM – they participate in submarket j up to the point where

$$\min\{1, \sigma_j\} \left[u(q_j) - \hat{z}_m^j - \hat{z}_b^j\right] - \iota \hat{z}_m^j - s\hat{z}_b^j = 0.$$

In *SM*, matching probabilities for buyers and sellers are $\alpha(\sigma_m) = \min\{1, \sigma_m\}$ and $\alpha(\sigma_m)/\sigma_m = \min\{1/\sigma_m, 1\}$. Tightness $\sigma_m > 1$ is inconsistent with equilibrium because we can increase sellers' expected surplus, $\min\{1/\sigma_m, 1\} [\hat{z}_m^m - c(q_m)]$, by attracting additional buyers without harming those who are already there. Similarly, $\sigma_m < 1$ is inconsistent with equilibrium since we can raise sellers' expected surplus by attracting fewer buyers but asking for a higher payment. Hence, $\sigma_m = 1$ and $\mu_m = n_m$. From the buyer's participation constraint, $\hat{z}_m^m = u(q_m)/(1 + \iota)$. Substituting this into the seller's surplus and maximizing wrt q_m, we get $u'(q_m)/c'(q_m) = 1 + \iota$, the usual condition except that now buyers trade with probability 1 because of the special matching technology.

In *S2*, following the above reasoning, $\sigma_2 = 1$ and $\mu_2 = n_2$. Again, if $z_m^2 > 0$ then $\rho = 0$, $q_2 = q_m$ and the submarkets are essentially identical. This regime is an equilibrium iff $z_b = A_b/n_2 \leq u(q_m)/(1 + \iota)$. If instead $z_m^2 = 0$ then q_2 solves $u'(q_2)/c'(q_2) = 1 + s$ and $z_b = A_b/n_2 = u(q_2)/(1 + s)$ makes buyers indifferent. Then $q_2 = q^*$ obtains if $s = 0$, which requires $A_b/n_2 \geq u(q^*)$. If $u(q_m)/(1 + \iota) < A_b/n_2 < u(q^*)$ then $q_2 < q^*$ and $s \in (0, \iota)$. So the outcome is like Figure 4 from the model with random search, instead of Figure 9, because $\sigma_m = \sigma_2 = 1$

and q_m are independent of z_b with this special matching technology. However, in this version buyers know which assets are accepted, as payment instruments or collateral, in their submarket of choice.[28]

5.3 Information and Segmentation

The final exercise if to endogenize the measures of sellers across submarkets with posting by assuming, as in Section 4.1, that they can recognize and hence accept bonds if they pay κ, again with CDF $F(\kappa)$. So a seller then chooses to participate in $S2$ if $\kappa \leq \Delta = \Pi_2 - \Pi_m$, and $n_2 = F(\Delta)$, similar to Section 4.1. Figure 10 plots Δ as a function of n_2 for an example (similar to the one in fn. 27). For an arbitrary $F(\kappa)$, equilibrium is unique, and an increase in A_b raises n_2 by shifts the relationship up. Uniqueness here contrasts with Section 4.1 because, when market makers posts the terms of trade, they internalize the strategic complementarities between sellers' information acquisition and buyers' portfolio decisions.[29]

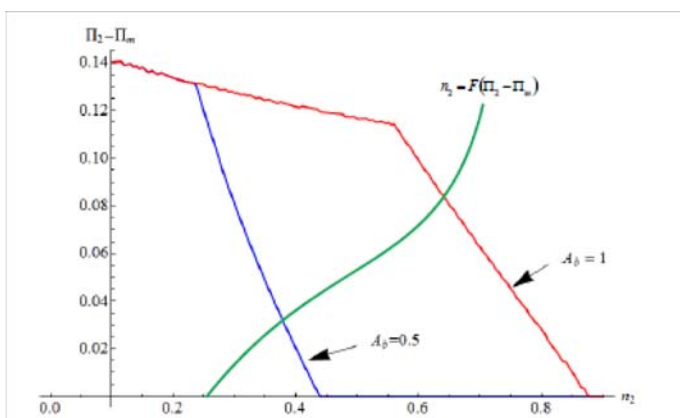

Figure 10: Information and segmentation

[28] One can also consider the case where the population of buyers is smaller, $\mu < n_m + n_2$. Then at least one of the submarkets has a shortage of buyers, $\mu_j < n_j$, and $\Pi_j = z_j - c(q_j) = 0$. If both markets have $\mu_j < n_j$, then $\Pi_m = \Pi_2 = 0$, and buyer indifference holds iff $s = \iota$. In this case $U^b = \max_{q \geq 0} \{-(1+\iota)c(q) + u(q)\}$. This regime is consistent with equilibrium iff $A_b \leq n_2 c(q)$ where q solves $u'(q)/c'(q) = 1 + \iota$. If $s < \iota$, then $\Pi_2 > \Pi_m = 0$ for buyers to be indifferent between the submarkets. Moreover, $q_2 > q_m$ and $\sigma_2 = 1 < \sigma_m = n_m/(\mu - n_2)$.

[29] For related arguments about the role of market makers, in different contexts, see Rocheteau and Wright (2005) or Faig and Huangfu (2007).

Also, equilibrium with $\iota = s$ and $\rho = 0$ now cannot exist if $F(0) = 0$ – i.e., if it is costly for any sellers to acquire the requisite information, none choose *S2* when $\Pi_2 = \Pi_m$. However, if $F(0) > 0$ then there is an equilibrium with $\iota = s$ and $n_2 = F(0)$ when A_b is small. The outcome looks Figure 9.[30] Specifically, for low A_b sellers participate in *S2* iff $\kappa = 0$, in which case $\iota = s$ and $\rho = 0$, so the two assets are perfect substitutes. As A_b increases, Δ becomes positive and some sellers with $\kappa > 0$ join *S2*. Output in both markets increases, but it increases more in *S2*. The reason q_m increases is because tightness in *SM* rises. In contrast, q_2 increases because the interest rate on bonds goes up, even though σ_2 decreases. As before, OMO's affect the composition of buyers and sellers and hence the matching probabilities in the submarkets, providing a channel through which the effects A_b spill over from across submarkets.

5.4 Other Segmentation Models

The earlier literature mentioned in the Introduction on segmented markets and monetary policy is very different. Those papers all have CIA constraints, and model segmentation by assuming that not all agents are active in some asset markets because of transactions costs, generally, or sometimes a fixed cost to transfer resources between asset and goods markets. They are interested in getting temporary changes in real interest rates and economic activity when the money supply increases, as well as a negative relation between expected inflation and real interest rates, or persistent liquidity effects on interest and exchange rates. Sometimes they try to integrate quantity-theory-like effects into models where policy is modeled by Taylor rules. Our approach is only tangentially related, despite a similar focus on segmented markets.

Buyers in our model endogenously choose to participate in particular markets,

[30] For constructing examples, we assumed 20% of sellers can authenticate bonds at no cost while the rest have κ uniformly distributed on $(0, 0.4)$. The other parameters are as in the previous example.

with neither transaction costs for participation nor for exchanging money and bonds. Moreover, the use of assets in the exchange process is motivated by explicit frictions concerning limited commitment/enforcement and information. A big difference between this and the CIA approach can be seen throughout the paper whenever we mention events like "a type-i buyer meets a type-j seller" or "a buyer chooses to look for a seller in submarket-j." There is no notion of who trades with whom in those models, as agents trade only against their augmented (e.g., by CIA) budget equations. The framework adopted here allows us to bring to bear standard tools from search-and-bargaining theory, including directed search. While it is obviously good for the profession to explore various alternatives, and while this earlier work is highly relevant, we also think that it is useful to model the exchange process in more detail when analyzing liquidity.[31]

6 Conclusion

We studied economies with money and bonds with a focus on conventional OMO's. In the model, agents that can be interpreted as consumers or producers trade in decentralized markets, where assets can be essential as either media of exchange or collateral. We grounded differences in asset liquidity on information frictions, and considered various specifications, including random or directed search, and bargaining or posting. With no *ad hoc* price rigidities, changing the level of the money supply is neutral. Hence, to understand OMO's, we concentrated on the effects of changing the stock of outstanding bonds. Theory delivers sharp predictions for these effects, even when there are multiple equilibria, and generates novel outcomes like negative nominal rates, market segmentation and liquidity traps. In terms of positive economics, buying back bonds with newly-

[31] In future research, it may be interesting to use some version of the directed search model in Guerrieri et al. (2010), which is designed explicity to deal with adverse selection concerning (among other things) asset quality. Chang (2012), Shao (2013) and Guerrieri and Shimer (2014) make some progress along these lines, but more can be done.

issued currency lowers the interest rate on T-bills, again, not by putting more cash in the hands of the public, but by contracting their holdings of liquid bonds. In terms of normative economics, this policy can reduce welfare for the obvious reason that it involves hoarding liquid assets (say, burying them in Fort Knox), whence people cannot use them for transactions purposes. In general, however, the welfare effects are unclear, since in the baseline model q_b and q_2 move in one and direction and q_m in the other, while in the directed search model the q's move in the same direction but matching probabilities are also affected.

Despite stark differences in modeling strategy, the basic predictions concerning the impact of OMO's on interest rates are consistent with 'convention wisdom' as often taught to undergraduates, discussed in policy circles, and inferred by some economists from the data. Of course, in the short run, there may be additional effects that our model is not designed to capture, involving a plethora of phenomena such as signal extraction or other information problems and distributional considerations. As regards the latter, in particular, recent contributions by Chiu and Molico (2014) and Jin and Zhu (2014) demonstrate how distributional effects generate dynamic responses to monetary injections that, for some parameterizations, resemble the data, in the sense that output rises quickly and prices slowly. And they do this with no exogenous restrictions on price flexibility – indeed, the real terms of trade are negotiated in every transaction, and it is precisely because output rises that prices *look* sticky, even though they are not, and hence nominal rigidities are not driving the results. As interesting as this may be, these kind of results require computational methods; by abstracting from distributional issues, we were able to prove rather a lot analytically.

Regarding interest rates in the 'real world,' one big message is that it is important to be clear *which* interest rates one has in mind. We focused on two, ι for illiquid and ρ for liquid assets, but one can easily imagine extending this to many assets, and hence many interest rates, based on the discussions in Lester et al. (2012) and Li et al. (2013). See also Venkateswarany and Wright (2013), where in addition to some theory one can find a fully-calibrated macro model where liquidity and pledgeability have leading roles. Another general message is that the impact of increasing the monetary supply can depend crucially on the it is injected into the economy, as emphasized recently by Wallace (2014). While our model represents just one facet of this idea, we learned a lot by studying increases in the money supply accomplished not by lump sum transfers, as often assumed in the theoretical literature, but by buying back government debt. There is much more to be done. As mentioned above, potential extensions include more general treatments of the labor market, alternative versions of asset markets with directed search and private information, models where the role of banking is made explicit, and specifications with private as well as government securities. All of this is left for other research.

Appendix: Additional Results for the Directed Search Model

Since in Section 5.2 we focused mainly on examples, here we present a more general directed search model with posting when there is one asset z, with a spread s between the return on it and on an illiquid bond; a special case is fiat money where $s = \iota$. Market makers post $(q, \hat{z}, \sigma)$ to solve a version of (24), with s instead of ι and Π instead of Π_m. Generically there is a unique solution, with $U^b(s, \Pi)$ decreasing in both s and Π (we assume Π is not too big, so the market can open). The FOC's wrt q and σ are given by (25)-(26) except with s and Π instead of ι and Π_m. This generates a correspondence $\sigma(\Pi)$, similar to a demand correspondence, with σ the quantity and Π the price, and one can show $\sigma(\Pi)$ is decreasing (Rocheteau and Wright 2005, Lemma 5).

Let us normalize the measure of buyers to $\mu = 1$. One approach in the literature assumes that n is fixed, and therefore in equilibrium $\sigma = n$ (the seller-buyer ratio in the representative submarket is the population ratio). Then $\sigma(\Pi) = n$ pins down Π. In this case,

$$\frac{\partial q}{\partial s} = \frac{c'}{\alpha u'' - (\alpha + s) c''} < 0, \text{ and } \frac{\partial q}{\partial n} = -\frac{\alpha' (u' - c')}{\alpha u'' - (\alpha + s) c''} > 0.$$

Also, suppose ε is constant, as it is with a Cobb-Douglas matching function (truncated to keep probabilities between 0 and 1). Then is easy to derive

$$\frac{\partial \hat{z}}{\partial s} = \frac{\alpha \{u'c' [\alpha + s(1-\varepsilon)] - \varepsilon(1-\varepsilon)(u-c) [\alpha u'' - (\alpha + s) c'']\}}{[\alpha + s(1-\varepsilon)]^2 [\alpha u'' - (\alpha + s) c'']} < 0$$
$$\frac{\partial \hat{z}}{\partial n} = \frac{\iota \alpha' \{\varepsilon(1-\varepsilon)(u-c) [\alpha u'' - (\alpha + s) c''] - u'c' [\alpha + s(1-\varepsilon)]\}}{[\alpha + s(1-\varepsilon)]^2 [\alpha u'' - (\alpha + s) c'']} > 0.$$

Another approach in the literature assumes a perfectly-elastic supply of homogeneous sellers, with fixed cost of entry κ, so that in equilibrium $\Pi = \kappa$ and $\sigma = \sigma(\kappa)$ is endogenous. In this case,

$$\frac{\partial q}{\partial s} = \frac{c'\alpha''(u-c)}{D} < 0, \text{ and } \frac{\partial q}{\partial \kappa} = -\frac{\alpha'[1 + s(1-\varepsilon)/\alpha](u' - c')}{D} < 0.$$

with $D = [\alpha u'' - (\alpha + s) c''] [\alpha''(u-c) + s\kappa(1-\varepsilon)\alpha'/\alpha^2] - \alpha'^2(u'-c')^2 > 0$ (while

D cannot be signed globally, except in special cases like $\iota = 0$, in equilibrium $D > 0$ by the SOC's). Also, if ε is constant, then

$$\frac{\partial \sigma}{\partial s} = \frac{[\alpha u'' - (\alpha + s)c'']\kappa(1-\varepsilon)/\alpha - \alpha'(u' - c')c'}{D} < 0$$
$$\frac{\partial \sigma}{\partial \kappa} = \frac{[\alpha u'' - (\alpha + s)\, c''][1 + s(1-\varepsilon)/\alpha]}{D} < 0$$
$$\frac{\partial \hat{z}}{\partial s} = \frac{\kappa(1-\varepsilon)^2 \left[\alpha u'' - (\alpha + s)\, c''\right] + c'^2 \left[\alpha(u - c)\alpha'' - s\kappa(1-\varepsilon)\alpha'\right]}{\alpha^2 D} < 0$$
$$\frac{\partial \hat{z}}{\partial \kappa} = -\frac{\iota\alpha' \left\{u' \left[\alpha + s(1-\varepsilon)\right] + \varepsilon(1-\varepsilon)c \left[\alpha u'' - (\alpha + s)\, c''\right]\right\}}{\alpha \left[\alpha + s(1-\varepsilon)\right] D} \gtreqless 0.$$

All of these are fairly intuitive.

To briefly mention efficiency, the FOC's imply $q = q^*$ iff $s = 0$. With entry, $s = 0$ also implies $\alpha'(\sigma)[u(q) - c(q)] = \kappa$. Hence, $s = 0$ implies $\sigma = \sigma^*$, where (q^*, σ^*) solves the planner problem $\max_{q,\sigma} \{\alpha(\sigma)[u(q) - c(q)] - \sigma\kappa\}$. For comparison, with Kalai bargaining, $s = 0$ implies $q = q^*$, but $\sigma = \sigma^*$ iff $1 - \theta = \varepsilon(\sigma^*)$, which is the Hosios (1990) condition, saying that bargaining shares should equal the elasticity of matching wrt participation. Since directed search yields (q^*, σ^*) automatically at $s = 0$, it is sometimes said that it satisfies the Hosios condition endogenously.

References

[1] G. Afonso and R. Lagos (2013) “Trade Dynamics in the Market for Federal Funds,” Econometrica, forthcoming.

[2] F. Alvarez, R. Lucas and W. Weber (2001) “Interest Rates and Inflation,” AEA Papers and Proceedings, 219-226.

[3] F. Alvarez, A. Atkeson and P. Kehoe (2002) “Money, Interest Rates, and Exchange Rates with Endogenously Segmented Markets,” JPE 110, 73-112.

[4] F. Alvarez, A. Atkeson and C. Edmond (2009) “Sluggish Response of Prices and Inflation to Monetary Shocks in an Inventory Model of Money Demand,” QJE.

[5] D. Andolfatto (2013) “Incentive-Feasible Deflation,” JME 60, 383-390.

[6] S. Aruoba, G. Rocheteau and C. Waller (2007) “Bargaining and the Value of Money,” JME 54, 2636-55.

[7] S. Aruoba, C. Waller and R. Wright (2011) “Money and Capital,” JME 58, 98-116.

[8] M. Bech and C. Monnet (2014) “Sorting in the Interbank Market,” mimeo.

[9] A. Berentsen, A. Marchesiani and C. Waller (2014)“Floor Systems for Implementing Monetary Policy: Some Unpleasant Fiscal Arithmetic,” RED, forthcoming.

[10] A. Berentsen, G. Menzio and R. Wright (2011) “Inflation and Unemployment in the Long Run,” AER 101, 371-98.

[11] A. Berentsen and C. Monnet (2008) “Monetary Policy in a Channel System,” JME 55, 1067-08.

[12] A. Berentsen and G. Rocheteau (2004) “Money and Information,” RES 71, 915-44.

[13] A. Berentsen and C. Waller (2011) “Price-Level Targeting and Stabilization Policy,” JMCB 43, 559-80.

[14] BIS (2001) "Collateral in Wholesale Financial Markets: Recent Trends, Risk Management and Market Dynamics,” Report prepared by the Committee on the Global Financial System Working Group on Collateral.

[15] R. Caballero (2006) “On the Macroeconomics of Asset Shortages,” NBER wp 12753.

[16] R. Caballero and A. Krishnamurthy (2006) “Bubbles and Capital Flow Volatility: Causes and Risk Management,” JME 53, 35-53.

[17] B. Chang (2012) "Adverse Selection and Liquidity Distortion in Decentralized Markets," mimeo.

[18] J. Chiu and M. Molico (2014) "Short-Run Dynamics in a Search-Theoretic Model of Monetary Exchange," mimeo.

[19] J. Chiu and C. Monnet (2014) "Relationships in the Interbank Market," mimeo.

[20] M. Dong (2011) "Inflation and Unemployment in Competitive Search Equilibrium," MD 15, 252-68.

[21] M. Dong and S. Xiao (2013) "Liquidity, Monetary Policy and Unemployment," mimeo.

[22] R. Dutu and S. Huangfu and B. Julien (2011) "Contingent Prices And Money," IER 52, 1291-308.

[23] M. Faig and S. Huangfu (2007) "Competitive-Search Equilibrium in Monetary Economies," JET 136, 709-18.

[24] M. Faig and B. Jerez (2006) "Inflation, Prices, and Information in Competitive Search," BEJ Macro 6.

[25] I. Fisher (1930) *The Theory of Interest*.

[26] M. Friedman (1968) "The Role of Monetary Policy," AER 58, 1-17.

[27] M. Galenianos and P. Kircher (2008) "A Model of Money with Multilateral Matching," JME 55, 1054-66.

[28] A. Geromichalos, J. Licari and J. Lledo (2007) "Asset Prices and Monetary Policy," RED 10, 761-79.

[29] A. Geromichalos, L. Herrenbrueck and K. Salyer (2013) "A Search-Theoretic Model of the Term Premium," mimeo.

[30] G. Gorton and G. Ordonez (2014) "Collateral Crisis," AER 104, 343-378.

[31] P. Gourinchas and O. Jeanne (2012) "Global Safe Assets," mimeo.

[32] C. Gu, F. Mattesini and R. Wright (2014) "Money and Credit Redux," mimeo.

[33] V. Guerrieri, R. Shimer and R. Wright (2010) "Adverse Selection in Competitive Search Equilibrium," Econometrica 78, 1823-1862.

[34] V. Guerrieri and R. Shimer (2014) "Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality," AER, forthcoming.

[35] H. Han (2014) "Over-the-Counter Markets, Intermediation and Monetary Policies," mimeo.

[36] P. He, L. Huang and R. Wright (2008) "Money, Banking and Monetary Policy," JME 55, 1013-24.

[37] A. Head, L. Liu, G. Menzio and R. Wright (2012) "Sticky Prices: A New Monetarist Approach," JEEA 10, 939-73.

[38] A. Hosios (1990) "On the Efficiency of Matching and Related Models of Search and Unemployment," RES 57, 279-98.

[39] T. Hu, J. Kennan and N. Wallace (2009) "Coalition-Proof Trade and the Friedman Rule in the Lagos-Wright Model," JPE 117, 116-137.

[40] S. Huangfu (2009) "Competitive Search Equilibrium with Private Information on Monetary Shocks," BEJ Macro 9, 1-27.

[41] G. Jin and T. Zhu (2014) "Non-neutrality of Money in Dispersion: Hume Revisited," mimeo.

[42] IMF (2012) The Quest for Lasting Stability (Global Financial Stability Report)

[43] E. Kalai (1977) "Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons," Econometrica 45, 1623-30.

[44] T. Kehoe and D. Levine (1993) "Debt-Constrained Asset Markets," RES 60, 865-88.

[45] A. Khan (2006) "The Role of Segmented Markets in Monetary Policy," FRB Phila. Business Review Q4.

[46] N. Kiyotaki and J. Moore (1997) "Credit Cycles," JPE 105, 211-48.

[47] N. Kiyotaki and J. Moore (2005) "Liquidity and Asset Prices," IER 46, 317-49.

[48] N. Kiyotaki and R. Wright (1989) "On Money as a Medium of Exchange," JPE 97, 927-54.

[49] N. Kiyotaki and R. Wright (1993) "A Search-Theoretic Approach to Monetary Economics,"AER 83, 63-77.

[50] R. Lagos and R. Wright (2005) "A Unified Framework for Monetary Theory and Policy Analysis," JEP 113, 463-44.

[51] R. Lagos and G. Rocheteau (2005) "Inflation, Output, and Welfare," IER 46, 495-522.

[52] R. Lagos and G. Rocheteau (2008) "Money and Capital as Competing Media of Exchange," JET 142, 247-258.

[53] R. Lagos, G. Rocheteau and R. Wright (2014) "The Art of Monetary Theory: A New Monetarist Perspective," mimeo.

[54] B. Lester, A. Postlewaite and R. Wright (2012) "Liquidity, Information, Asset Prices and Monetary Policy," RES 79, 1209-38.

[55] L. Liu, L. Wang and R. Wright (2014) "Costly Credit and Sticky Prices," mimeo.

[56] Y. Li, G. Rocheteau and P. Weill (2012) "Liquidity and the Threat of Fraudulent Assets," JPE 120, 815-46.

[57] R. Lucas (1972) "Expectations and the Neutrality of Money," JET 4, 103-24.

[58] E. Moen (1997) "Competitive Search Equilibrium," JPE 105, 385-411.

[59] D. Mortensen and R. Wright (2002) "Competitive Pricing and Efficiency in Search Equilibrium," IER 43, 1-20.

[60] R. Mundell (1963) "Inflation and Real Interest," JPE 71, 280-283.

[61] G. Rocheteau and A. Rodriguez-Lopez (2013) "Liquidity Provision, Interest Rates, and Unemployment," JME, in press.

[62] G. Rocheteau, P. Rupert, K. Shell and R. Wright (2008) "General Equilibrium with Nonconvexities and Money," JET 142, 294-317.

[63] G. Rocheteau, P. Rupert and R. Wright (2007) "Inflation and Unemployment in General Equilibrium," Scand J Econ 109, 837-55.

[64] G. Rocheteau and R. Wright (2005) "Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium," Econometrica 73, 175-202.

[65] D. Sanches and S. Williamson (2010) "Money and Credit with Limited Commitment and Theft," JET 145, 1525-49.

[66] E. Shao (2013) "The Threat of Counterfeiting in Competitive Search Equilibrium," mimeo.

[67] R. Silveira and R. Wright (2010) "Search and the Market for Ideas" JET 145, 1550-73.

[68] Swiss National Bank (2013) *106th Annual Report.*

[69] V. Venkateswarany and R. Wright (2013) "A New Monetarist Model of Financial and Macroeconomic Activity," NBER Macro Annual.

[70] N. Wallace (1981) "A Modigliani-Miller Theorem for Open-Market Operations," AER 71, 267-74.

[71] N. Wallace (1983) "A Legal Restriction Theory of the Demand for Money and the Role of Monetary Policy," FRB Minneapolis QR 7.

[72] N. Wallace (2010) "The Mechanism Design Approach to Monetary Theory," Handbook of Monetary Economics, vol. 2, B. Friedman and M. Woodford, eds. North-Holland, Amsterdam.

[73] N. Wallace (2014) "Optimal Money-Creation in 'Pure-Currency' Economies: A Conjecture," QJE 129, 259-275.

[74] S. Williamson (2012) "Liquidity, Monetary Policy, and the Financial Crisis: A New Monetarist Approach," AER 102, 2570-605.

[75] S. Williamson (2014*a*) "Scarce Collateral, the Term Premium, and Quantitative Easing," mimeo.

[76] S. Williamson (2014*b*) "Central Bank Purchases of Private Assets," mimeo.

[77] S. Williamson and R. Wright (1994) "Barter and Monetary Exchange under Private Information," AER 84, 104-23.

[78] S. Williamson and R. Wright (2010*a*) "New Monetarist Economics I: Methods," FRB St. Louis Review.

[79] S. Williamson and R. Wright (2010*b*) "New Monetarist Economics I: Models," Handbook of Monetary Economics, vol. 2, B. Friedman and M. Woodford, eds. North-Holland.

[80] R. Wong (2012) "A Tractable Monetary Model under General Preferences," mimeo.

[81] R. Wright (2010) "A Uniqueness Proof for Monetary Steady State," JET 145, 382–91.