Central Bank Digital Currency: a Corporate Finance Perspective*

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Abstract

We build models with an interest-bearing central bank digital currency (CBDC) to investigate the impacts of issuing CBDC on banking and the macroeconomy. When CBDC is the only asset, a higher interest rate on CBDC does not necessarily lead to financial disintermediation. It could promote bank lending and firm investment because CBDC and bank deposits are complements. The interest rate on reserves and the required reserve ratio can affect bank lending and investment when the reserve constraint binds. In our extensions, cash and interest-bearing CBDC can coexist, where the coexistence may require the central bank to adjust either the CBDC interest rate or the interest rate on reserves. Our results suggest that the design of CBDC and banking matters for understanding how CBDC affects the macroeconomy.

Key words: CBDC, money, banking, interest rates, investment

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1 Introduction

The current paper money system has been challenged by private cryptocurrencies since Bitcoin was created in 2009, based on a blockchain technology.[1] Recently, stablecoins such as Libra (now Diem) and Tether (USDT) aim to minimize the huge price volatility of Bitcoin-like cryptocurrencies, by maintaining relatively stable exchange rates with major currencies like the US dollars and others.[2] All these private cryptocurrencies have generated great concerns among central banks and policy makers around the world, and pushed them to explore the possibility of introducing a central bank digital currency (CBDC). The COVID-19 pandemic has further expedited this progress, due to the concern that cash payments could carry virus.

By Oct. 2020, a survey conducted by Bank for International Settlements found that "*80% of central banks are engaged in investigating CBDC and half have progressed past conceptual research to experimenting and running pilots*" (BIS, 2020). The People's Bank of China may be the pioneer in experimenting CBDC (called DC/EP, digital currency and electronic payment), and has run pilot projects in multiple cities since 2020. The pilot projects have revealed major features of DC/EP: first, it belongs to M0, the same as cash in circulation, except in a digital form; second, it is distributed through a two-tier system, where the central bank stays at the first tier, with commercial banks and non-bank financial institutions at the second tier; third, it aims to achieve "controllable anonymity", but will coexist with cash for a long time.[3]

Given the forthcoming CBDC, our paper will address these questions: what are the

[1] By early 2021, there are more than 4000 types of cryptocurrencies in total, and the market value of Bitcoin (the top cryptocurrency) has reached US$ 1 trillion and that of Ethereum (top 2) is around US$ 172 billion.

[2] On June 18, 2019, Facebook and its partners issued the first white paper of Libra, which is a new cryptocurrency with the mission "to enable a simple global currency and financial infrastructure that empowers" over 2.7 billion Facebook users. It will still use the blockchain technology, but the design is a "stablecoin" which aims to minimize price volatility, with the full backup of reserves from a basket of multiple fiat currencies and credible government securities. Compared to Bitcoin-like cryptocurrencies, these features make Libra more possible to serve as a "currency", i.e., serving as a medium of exchange, a unit of account and a store of value.

[3] "Controllable anonymity" means commerical banks and non-banks (AliPay, Wechat Pay and three telecom operators) at the second tier are responsible for compliance on users' data privacy, while the central bank has all payment data for backup (Zhou, 2020).

impacts of CBDC on financial intermediation, firm investment and the macroeconomy? Since CBDC can potentially bear interest, will it cause financial disintermediation? How will CBDC affect the conduct of monetary policy? How can interest-bearing CBDC coexist with cash? What are the implications for monetary policy when they coexist?

To answer the above questions, we start from a benchmark model with only CBDC, where we explicitly model a frictional deposit market and a frictional loan market to incorporate banking and investment. Entrepreneurs hold CBDC, and may or may not have investment opportunities. If they do not have investment opportunities (labeled as type-0), they deposit idle CBDC at banks in a frictional deposit market. If they do (labeled as type-1), they use CBDC as a down payment to apply for bank loans in a frictional loan market, to acquire capital and produce final output. We consider *four* policy tools. One is a traditional monetary policy tool of changing the growth rate of money supply (equivalent to changing the inflation rate at steady states). The second is a new tool of changing the CBDC interest rate. Banks in our model are subject to a reserve requirement, but the central bank pays interest to reserves. Hence, we can consider two additional policy tools: changing the reserve ratio and changing the interest rate on reserves.

There are *two* main results from the benchmark model. The *first* result is that a higher CBDC interest rate tends to have a positive impact on bank lending and investment, which implies that CBDC does not necessarily lead to financial disintermediation. This result is in sharp contrast with findings in existing models of CBDC. For example, in Andolfatto (2018) and Keister and Sanches (2020), CBDC and bank deposits are substitutes. A higher CBDC interest rate crowds out deposits, which reduces bank lending and investment. An exception is in Chiu et al. (2021) where CBDC and bank deposits are substitutes, but owing to the imperfect competition in the deposit market, a higher CBDC interest rate may help limit banks' market power and force banks to offer a higher deposit rate to prevent depositors from switching bank deposits to CBDC. Therefore, deposits and loans increase in response to the higher CBDC interest rate. In their model, the CBDC interest rate serves as a floor for the deposit rate.

The critical difference between our model and these existing models is that CBDC and banks are complements in the spirit of Berentsen et al. (2007).[4] Banks help channel idle liquidity from type-0 to type-1 entrepreneurs. The complementarity between CBDC and bank deposits makes a higher CBDC interest rate more favorable to deposits and investment. In the two-tier system of China's CBDC, CBDC wallets/accounts are opened through the second tier (mainly commercial banks), and are linked with bank accounts to facilitate the conversion between CBDC and bank deposits. A critical purpose of the two-tier system is to avoid financial disintermediation. Our results lend support to this system. Therefore, the key message from the benchmark model is that the relationship between CBDC and banking matters when it comes to assessing the macroeconomic effects of CBDC.

The *second* result is that the interest rate on reserves and the reserve requirement ratio can be independent monetary policy tools in the equilibrium where the reserve constraint binds. When the reserve constraint does not bind, the interest rate on reserves and the reserve ratio do not affect the general equilibrium allocation. The binding reserve constraint implies that both the interest rate on reserves and the reserve ratio can directly affect the amounts of loans being issued as well as investment. A higher interest rate on reserves or a lower reserve ratio makes entrepreneurs hold less CBDC, but allows banks to issue more loans. In practice, the interest rate on reserves is usually used by central banks as a lower bound in the channel system or floor system. We find that it has a potential role in affecting bank lending and investment through the reserve constraint in our model.

To understand how cash and CBDC interact, we extend the benchmark model by adding cash to the portfolio of entrepreneurs.[5] This aims to capture, at least, the initial stage of issuing CBDC, when it is more realistic that cash and CBDC coexist. We consider two special scenarios. In the first scenario, banks can accept only cash as deposits and use it as reserves. Banks can help entrepreneurs to store CBDC, but cannot use it as reserves, which

[4] Our paper and the current CBDC literature represent two main ways to model money and banking, as summarized in the survey by Lagos et al. (2017), "in some of these, money and banking are complements, since a bank is where one goes to get cash; in others, they are substitutes, since currency and bank liabilities are alternative payment instruments, allowing one to discuss not only currency but also checks or debit cards".

[5] Our paper focuses on entrepreneur's cash holding because corporate cash holding has been an important issue for firms in the U.S. and other advanced economies since the 1980s (see Bates et al. 2009, Azar et al. 2015, Graham and Leary 2018, among others). Graham and Leary (2018) document that the level of average cash holdings is around 25% of assets for U.S. firms.

follows the assumptions in Andolfatto (2018). In the second scenario, banks can accept only CBDC as deposits and use it into reserves. This captures the features of fast-growing new types of banks, i.e., Internet banks or the online banking business operated by traditional banks, where banks mainly deal with digital/electronic money, and do not accept cash.

In both scenarios, cash and CBDC can coexist in all general equilibria, even when CBDC has a non-zero interest rate. In some equilibria, the coexistence requires the central bank to give up either the CBDC interest rate or the interest rate on reserves as an independent policy tool. When the reserve constraint binds, cash and CBDC can coexist without the need to sacrifice any policy tools. Our results on coexistence demonstrate how coexistence can be achieved by considering economic tradeoffs between cash and CBDC without the need to assume limited participation or segmented markets. The extensions also highlight the importance of the relationship between CBDC and banking in understanding the effects of CBDC. In the first scenario where cash and banking are complements, a higher CBDC interest rate does cause financial disintermediation as entrepreneurs switch from cash to CBDC and less cash holdings reduce bank lending. However, in the second scenario, CBDC and banking become complements and our result confirms the finding in the benchmark model that CBDC may not cause financial disintermediation.

Our paper is related to *three* lines of literature. The first line is the literature on CBDC, including Andolfatto (2018), Keister and Sanches (2020), Chiu et al. (2021), and many policy reports on CBDC including but not limited to Bordo and Levin (2017) and Berentsen and Schar (2018). Keister and Sanches (2020) build a model where both central bank money and private bank deposits are used in exchange, to study the effects of introducing CBDC on interest rates, economic activity and welfare. They have competitive banking, and model CBDC and bank deposits as substitutes. Their results show that introducing CBDC can promote efficiency in exchange and raise welfare, but also crowds out bank deposits and decreases investment. In contrast, with the setting of non-competitive banking, Andolfatto (2018) and Chiu et al. (2021) both study the impacts of issuing CBDC on banking. Their difference is that Andolfatto (2018) uses an OLG model with monopolistic banking, while Chiu et al. (2021) use a New Monetarist model with a competitive loan market, but a Cournot-oligopolistic deposit market.

In all three papers, CBDC and bank deposits are modelled as substitutes, whereas CBDC and deposits are complements in our model. This difference could reflect potentially different designs of CBDC and banking and sheds light on the design of China's CBDC. In addition, our model generates the coexistence of cash and interest-bearing CBDC without resorting to the assumption of limited participation or segmented markets, which are common assumptions to ensure coexistence.

The second line is the banking literature. There are many papers on banking since the canonical paper of Diamond and Dyvbig (1983).[6] Here we list a few that are highly related to our paper. Banks in our models accept idle liquidity as bank deposits and then make loans to those who need liquidity. This is the key mechanism to make CBDC and bank deposits become complements. The role of banks is similar to Berentsen et al. (2007). However, they focus on households' portfolios and the model does not have capital and investment. The model's banking sector is competitive. In contrast, we focus on corporate finance, where capital and investment are key choices, and the banking sector is frictional. Our paper shares a similar frictional loan market as Rocheteau et al. (2018b), which address the effects of monetary policy from a corporate finance perspective. However, we incorporate a frictional deposit market to link CBDC and banking.

The third line of literature is about cryptocurrency and the blockchain technology such as Chiu and Koeppl (2017) and Schilling and Uhlig (2018).[7] These papers help us understand cryptocurrency and the blockchain technology, particularly how cryptocurrencies are different from fiat money. Our paper differs from these papers since CBDC is not cryptocurrency per se.

The rest of the paper is organized as follows. Section 2 describes the environment. Section 3 introduces the benchmark model, where CBDC is the only asset. We present the policy analysis from the benchmark model in Section 4. Section 5 extends the benchmark model by adding cash, and considers two scenarios: cash only deposits and CBDC only deposits. Section 6 concludes the paper.

[6] Some recent papers that study banking include Williamson (2012), Gu et al. (2013), Brunnermeier and Sannikov (2016), Dong et al. (2021), etc.

[7] More references include Hendry and Zhu (2017), Huberman et al. (2017), Abadi and Brunnermeier (2018), Dong et al. (2019), Choi and Rocheteau (2020a, 2020b), etc.

2 Environment

Time is discrete and continues forever. Each period consists of three stages: Stage 1 is a decentralized deposit market; Stage 2 has a decentralized loan market, and a competitive capital market operating in parallel; and Stage 3 is a centralized market (CM). There are three types of agents: entrepreneurs (e), suppliers (s) and banks (b). There is a measure one of entrepreneurs, who are subject to an investment shock. With a probability n, $n > 1/2$, an entrepreneur has an investment opportunity and needs to acquire capital for production. With the remaining probability $1 - n$, the entrepreneur does not have an investment opportunity. We label them as type-1 and type-0 entrepreneurs, respectively. The investment shock is realized at the beginning of each period. Suppliers can provide capital in the capital market. As in Rocheteau et al. (2018b), the measure of suppliers is irrelevant due to constant returns. There is a measure one of banks that need to first take deposits in the deposit market, satisfying a reserve requirement, and then issue loans in the loan market. Banks are owned by all entrepreneurs equally.

In the benchmark model, we assume that a central bank issues only CBDC m_c. This describes the scenario when CBDC completely phases out paper money. We will consider the coexistence of cash and CBDC in Section 5. CBDC is a fiat digital money with the price ρ, measured by the CM numeraire goods x. It is interest-bearing with a nominal interest rate i_c paid every period. The timeline of a representative period is shown in Figure 1, and the details of each stage are as follows.

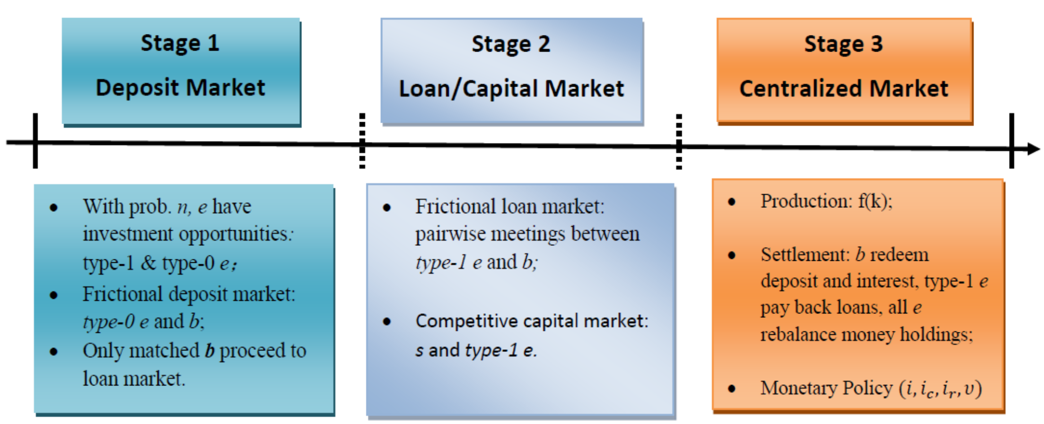

Figure 1: Timeline of a Representative Period

At Stage 1, all banks go to the deposit market to take deposits because banks need reserves to make loans in the subsequent loan market. After the investment shock is realized, all type-0 entrepreneurs go to the deposit market to deposit their idle balances. We assume a simple matching technology in the deposit market: short-side being served. Given the measure of type-0 entrepreneurs is $1-n$, the matching probability for a type-0 entrepreneur is 1 and that for a bank is $1-n$. Banks that do not get deposits will not proceed to the loan market, due to a reserve requirement which requires banks to hold a fraction υ $(0<\upsilon<1)$ of total assets in the form of reserves.[8] Banks and entrepreneurs bargain over the terms of the deposit contract. For simplicity, we assume banks make take-it-or-leave-it offers.

Banks obtaining deposits and all type-1 entrepreneurs participate in Stage 2. We again adopt the simple matching technology: short-side being served. Given that the measure of type-1 entrepreneurs is n and that of banks is $1-n$, the matching probability for a type-1 entrepreneur is $(1-n)/n$ and that for a bank is 1 since $n>1/2$. Banks and entrepreneurs bargain over the terms of the loan contract, including a down payment p (in the form of CBDC), a loan service fee ϕ and a loan size ℓ. Then the banked type-1 entrepreneurs can use the down payment plus the loan to acquire capital from suppliers at the market price q_k. To ensure the repayment of loans, we follow Rocheteau et al. (2018b) to assume that a fraction χ of the entrepreneur's output is pledgeable. It implies that banks can seize a fraction χ of output in the case of default. As for unbanked type-1 entrepreneurs, i.e., those who do not get loans from banks, they use internal finance to purchase capital from suppliers.

At Stage 3, all agents participate in the competitive CM. Entrepreneurs who deposit at Stage 1 redeem their deposits, and entrepreneurs who borrow in the loan market repay the loans including banking service fees. Banks distribute all profits to entrepreneurs. Entrepreneurs use capital k to produce, with the technology $f(k)$, where $f'(k)>0$, $f''(k)<0$ and $f(k)$ satisfies the Inada conditions. All agents can consume the numeraire good x in the CM. A negative x implies that agents work and produce x.

The government is active only at Stage 3. It is a consolidated monetary and fiscal authority, and the fiscal policy passively accommodates the monetary policy. Let M be the

[8] Notice that, to model the reserve requirement, Rocheteau et al. (2018b) introduce an interbank market where banks can borrow at a policy rate, while we introduce a deposit market.

amount of CBDC in the end of Stage 3 and (M_c, M_r) be the amounts of CBDC and reserves in the beginning of Stage 3 in the current period, where $M = (1 + i_c) M_c + (1 + i_r) M_r$ and (i_c, i_r) are the interest rates on CBDC and reserves. Note that M is only in the form of CBDC because there is no reserve in the economy in the end of each period because banks distribute all profits in Stage 3. We use the subscript "$-$" to denote variables associated with the previous period. The budget constraint of the government is,

$$G + T = \rho(M - M_{-}) - \rho i_c M_c - \rho i_r M_r, \tag{1}$$

where G is government spending and T is lump-sum transfers in real terms. The LHS in (1) refers to the total government expenditure, while the RHS is the seigniorage revenues net of CBDC and reserves interest payments. We consider *four* types of policy tools. The first one is to change the growth rate of CBDC, μ, measured by

$$\frac{M}{M_{-}} = 1 + \mu,$$

where $1+\mu \equiv \rho/\hat{\rho}$ in the steady state and μ is the inflation rate. The Fisher equation implies that $1 + i = (1 + \mu)/\beta$.[9] The second one is to set the interest rate of CBDC, i_c, with $i_c \leq i$ because of the no-arbitrage condition, but it is possible to have $i_c \geq 0$ or $i_c < 0$. When $i_c < 0$, the central bank implements a negative interest rate (NIR) policy. The third one is to change the interest rate of bank reserves, i_r, again with $i_r \leq i$ because of the no-arbitrage condition. When $i_r = 0$, the central bank pays zero interest to bank reserves, as in a typical floor system in normal times. When $i_r > 0$, reserves become interest-bearing. Paying interest on reserves has become a new monetary policy tool for a number of central banks including the Bank of England and the U.S. Federal Reserve since the Great Recession. The first three tools are monetary policy tools and the fourth one is a banking policy tool, where the central bank can change the required reserve ratio υ, $0 < \upsilon < 1$.

[9] Here i can be interpreted as the nominal interest rate of illiquid bonds, which measure the opportunity cost of holding fiat money with zero interest.

3 Model

In the benchmark model, we use U^j, V^j and W^j to denote the value functions for a type-j agent at Stages 1, 2 and 3, where $j = \{e, b, s\}$. For $j = e$, we have U_i^e, V_i^e and W_i^e for $i = \{0, 1\}$, to differentiate type-0 and type-1 entrepreneurs once the investment shock is realized in the beginning of Stage 1.

We start from Stage 3 in the current period, followed by Stages 1 and 2 in the next period. In the beginning of Stage 3, there are two types of entrepreneurs determined by the realized investment shock at Stage 1. For a type-1 entrepreneur,

$$W_1^e(z_c, \ell, k) = \max_{x, \hat{z}_c} \{x + \beta \mathbb{E} U^e(\hat{z}_c)\}$$
$$\text{st. } x + (1+\mu)\hat{z}_c = (1+i_c)z_c - \ell + f(k) + T + \Pi,$$

where $z_c = \rho m_c$ is the real balance of CBDC, $\hat{z}_c = \hat{\rho}\hat{m}_c$ is the real balance of CBDC carried to the next period, ℓ denotes the amount of loans incurred in the previous loan market, $f(k)$ is the final output with capital k, and (T, Π) represent transfers from the government and profits distributed by banks. Substituting x from the budget constraint, we have

$$W_1^e(z_c, \ell, k) = (1+i_c)z_c - \ell + f(k) + T + \Pi + \max_{\hat{z}_c}\{-(1+\mu)\hat{z}_c + \beta \mathbb{E} U^e(\hat{z}_c)\}.$$

For a type-0 entrepreneur,

$$\begin{aligned} W_0^e(z_c, d) &= \max_{x, \hat{z}_c} \{x + \beta \mathbb{E} U^e(\hat{z}_c)\} \\ \text{st. } x + (1+\mu)\hat{z}_c &= (1+i_c)z_c + (1+i_d)\, d + T + \Pi, \end{aligned}$$

where d is the real balance of deposits and i_d is the nominal deposit rate. The entrepreneur redeems deposits, but does not hold any capital or need to repay bank loans at Stage 3. Similarly, we have

$$W_0^e(z_c, d) = (1+i_c)z_c + (1+i_d)\, d + T + \Pi + \max_{\hat{z}_c}\{-(1+\mu)\hat{z}_c + \beta \mathbb{E} U^e(\hat{z}_c)\}.$$

It is clear that entrepreneurs will choose the same $\hat{z}_c$ independent of their previous types,

$$1+\mu=\beta\frac{\partial\mathbb{E}U^e\left(\hat{z}_c\right)}{\partial\hat{z}_c}. \tag{2}$$

Banks distribute their profits to entrepreneurs, where $\Pi=\sum\Pi_b$ aggregates all profits from active banks in this period. For each bank,

$$W^b\left(z_c,z_r,\ell,d\right)=\left(1+i_c\right)z_c+\left(1+i_r\right)z_r+\ell-\left(1+i_d\right)d+\beta U^b,$$

where (z_c,z_r) denote the real balances of CBDC and required reserves, ℓ is the loan repayment received from the type-1 entrepreneur and d is the deposit paid to the type-0 entrepreneur. We define $\Pi_b\equiv\left(1+i_c\right)z_c+\left(1+i_r\right)z_r+\ell-(1+i_d)d$.

For a supplier, $W^s=\omega+\beta V^s$,where ω is the wealth upon entering the CM, and V^s is the value function in the capital market at Stage 2 of next period (since suppliers are only active at Stage 2 and 3).

Moving to Stage 1 in the next period, after the investment shock is realized, type-1 entrepreneurs will directly proceed to the loan/capital market at Stage 2 and type-0 entrepreneurs enter the deposit market to deposit their idle balances. For entrepreneurs,

$$\mathbb{E}U^e\left(\hat{z}_c\right)=nU_1^e\left(\hat{z}_c\right)+\left(1-n\right)U_0^e\left(\hat{z}_c\right). \tag{3}$$

where $U_1^e(\hat{z}_c)=V_1^e\left(\hat{z}_c\right)$ and $U_0^e\left(\hat{z}_c\right)=W_0^e\left(\hat{z}_c-d,d\right)$ for $d\leq\hat{z}_c$. For banks, $U^b=\left(1-n\right)V^b\left(z_r,d\right)+nV^b\left(0,0\right)$, since banks who do not get deposits will exit from the market. That is, $V^b\left(0,0\right)=W^b\left(0\right)=0$, with $\Pi_b=0$. We will consider $z_r=d$ later where banks use all deposits as reserves to make loans.

In the loan market at Stage 2, type-1 entrepreneurs and banks with deposits meet. For a type-1 entrepreneur,

$$V_1^e\left(\hat{z}_c\right)=\frac{1-n}{n}W_1^e\left(\hat{z}_c-\frac{p_b}{1+i_c},\ell,k_b\right)+\frac{2n-1}{n}W_1^e\left(\hat{z}_c-\frac{p_m}{1+i_c},0,k_m\right).$$

With probability $(1-n)/n$, a type-1 entrepreneur successfully matches with a bank and uses down payment p_b, $p_b \leq (1+i_c)\hat{z}_c$ to get a loan ℓ to acquire capital k_b. The loan size ℓ satisfies $\ell = k_b - p_b + \phi$, which includes the amount of borrowing to purchase capital $k_b - p_b$ and the banking service fee ϕ. With the remaining probability, the entrepreneur can only resort to internal finance p_m, $p_m \leq (1+i_c)\hat{z}_c$, to acquire capital k_m. Notice the subscripts (b, m) denote terms related to banked and unbanked type-1 entrepreneurs. For a bank,

$$V^b(z_r, d) = -(k_b - p_b) + W^b(p_b, z_r, \ell, d), \tag{4}$$

where ℓ is the loan repaid by the entrepreneur. For suppliers in the capital market, $V^s = \max_k \{-k + W^s(q_k k)\}$,which leads to $q_k = 1$.

3.1 Bargaining

After defining the value functions, we consider how the deposit contract and the loan contract are determined. In the deposit market, let γ be the bargaining share of banks with $0 < \gamma \leq 1$. The Nash bargaining problem in the deposit market is

$$\max_{d, i_d} [\phi + (i_r - i_d) d]^\gamma [(i_d - i_c) d]^{1-\gamma} \text{ st. } d \leq \hat{z}_c. \tag{5}$$

Since we assume that banks make take-it-or-leave-it offers, we have $\gamma = 1$ and it gives $(i_d - i_c) d = 0$, which implies that $i_d = i_c$.[10] We let $d = \hat{z}_c$ as type-0 entrepreneurs are indifferent to depositing or not.

In the loan market, the surplus of a type-1 entrepreneur is

$$W_1^e\left(\hat{z}_c - \frac{p_b}{1+i_c}, \ell, k_b\right) - W_1^e\left(\hat{z}_c - \frac{p_m}{1+i_c}, 0, k_m\right) = f(k_b) - k_b - \phi - \Delta_m,$$

where $\Delta_m \equiv f(k_m) - p_m$ is the trading surplus for an unbanked type-1 entrepreneur and

[10] Given that bargaining in the deposit market occurs before bargaining in the loan market, the bargaining outcome in the deposit market could potentially affect the bargaining outcome in the loan market. We first solve for the bargaining solution in the loan market taking the deposit contract as given and then move back to solve the bargaining problem in the deposit market. The take-it-or-leave-it offer greatly simplifies the solution so that we have $i_d = i_c$ immediately.

$p_m = k_m = (1+i_c)\hat{z}_c$. The bank's surplus is ϕ. Let the bank's bargaining power be θ. Taking d and $\hat{z}_c$ as given, the Kalai bargaining problem is[11]

$$\max_{p_b,\phi,k_b} \phi$$

$$\text{st. } \phi = \theta \left[f(k_b) - k_b - \Delta_m \right], \tag{6}$$

$$k_b - p_b + \phi \leq \chi f(k_b), \tag{7}$$

$$k_b - p_b \leq \delta d (1 + i_r), \tag{8}$$

$$p_b \leq (1 + i_c)\hat{z}_c, \tag{9}$$

where we define the loan to reserve ratio $\delta \equiv 1/\upsilon - 1$. In what follows, we will use δ when discussing the banking policy as there is a one-to-one relationship between υ and δ. The first constraint (7) is the collateral constraint for the type-1 entrepreneur: he uses a fraction χ of final output $f(k)$ as the collateral to get bank loans. The second constraint (8) is the reserve constraint for the bank: the amount of lending is constrained by the amount of reserves (plus interest) held by the bank. Banks need to satisfy the reserve requirement. The third constraint (9) indicates the down payment p_b cannot exceed the real balances of CBDC plus interest. We let $p_b = (1 + i_c)\hat{z}_c$ because there is no benefit for the type-1 entrepreneur to keep some extra CBDC given that $i_c \leq i$.

We set up the Lagrangian,

$$\begin{aligned}\mathcal{L}(k_b, \lambda_1, \lambda_2) = &\max_{k_b,\lambda_1,\lambda_2} \theta \left[f(k_b) - k_b - \Delta_m \right] \\ &- \lambda_1 \left\{ k_b - (1 + i_c)\hat{z}_c + \theta \left[f(k_b) - k_b - \Delta_m \right] - \chi f(k_b) \right\} \\ &- \lambda_2 \left[k_b - (1 + i_c)\hat{z}_c - \delta (1 + i_r) d \right].\end{aligned}$$

[11] We adopt Kalai bargaining, instead of Nash, because the former is analytically more tractable. In previous versions, we try Nash bargaining and it does not change our main results.

The FOCs with respect to $(k_b, \lambda_1, \lambda_2)$ are

$$k_b : \theta[f'(k_b) - 1] = \lambda_1[(\theta - \chi)f'(k_b) + 1 - \theta] + \lambda_2$$

$$\lambda_1 : \lambda_1\{k_b - (1 + i_c)\hat{z}_c + \theta[f(k_b) - k_b - \Delta_m] - \chi f(k_b)\} = 0$$

$$\lambda_2 : \lambda_2[k_b - (1 + i_c)\hat{z}_c - \delta d(1 + i_r)] = 0,$$

where $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$. Hence, there are three cases to consider as follows.

Case 1: $\lambda_1 = 0$ and $\lambda_2 = 0$. Neither the collateral constraint nor the reserve constraint binds. We have $k_b = k^*$ with $f'(k^*) = 1$ and $\phi = \theta[f(k^*) - k^* - \Delta_m]$.

Case 2: $\lambda_1 > 0$ and $\lambda_2 = 0$. The collateral constraint binds but the reserve constraint does not. We have

$$(\theta - \chi)f(k_b) + (1 - \theta)k_b = k_m + \theta\Delta_m \tag{10}$$

$$\lambda_1 = \frac{\theta[f'(k_b) - 1]}{(\theta - \chi)f'(k_b) + 1 - \theta} > 0 \tag{11}$$

to solve for (k_b, λ_1) and ϕ solves (6).

Case 3: $\lambda_1 = 0$ and $\lambda_2 > 0$. The reserve constraint binds but the collateral constraint does not. We solve for (k_b, λ_2, ϕ) from

$$k_b = \delta d(1 + i_r) + k_m \tag{12}$$

$$\lambda_2 = \theta[f'(k_b) - 1] > 0, \tag{13}$$

and (6). Notice that a fourth case with $\lambda_1 > 0$ and $\lambda_2 > 0$ is not generically possible because both the collateral and reserve constraints can be used to solve for k_b for given d and $\hat{z}_c$.

3.2 General Equilibrium

The solutions from the deposit contract and the loan contract allow us to solve for $\hat{z}_c$ at Stage 3 using (2) and characterize the general equilibrium.

Definition 1 *Given policy parameters (G, i, i_c, i_r, δ), a stationary monetary equilibrium is a list of $(\hat{z}_c, k_b, \phi, i_d, d)$ that satisfies [1] bargaining solutions in the loan and deposit markets; [2] entrepreneurs' optimization; [3] the government budget constraint; and [4] all market clearing conditions.*

To determine $\hat{z}_c$, we first derive $\mathbb{E}U^e(\hat{z}_c)$ following (3)

$$\mathbb{E}U^e(\hat{z}_c) = (1-n)[f(k_b) - k_b - \phi - \Delta_m] + n[(1+i_c)\hat{z}_c - p_m + f(k_m)] + (1-n)[d(1+i_d) + (\hat{z}_c - d)] + nW_1^e(0,0) + (1-n)W_0^e(0,0).$$

With $p_m = \hat{z}_c(1+i_c) = k_m$, $d = \hat{z}_c$ and $i_d = i_c$, we have

$$\frac{\partial \mathbb{E}U^e(\hat{z}_c)}{\partial \hat{z}_c} = (1-n)(1-\theta)\{[f'(k_b)-1]\frac{\partial k_b}{\partial \hat{z}_c} - [f'(k_m)-1](1+i_c)\} + n[f'(k_m)-1](1+i_c) + (1+i_c). \tag{14}$$

To get $\partial k_b/\partial \hat{z}_c$, we consider the three cases to for the general equilibrium analysis.

In Case 1, $k_b = k^*$ and $\partial k_b/\partial \hat{z}_c = 0$. Using (2) and (14), we solve for k_m from

$$\frac{i - i_c}{1+i_c} = A[f'(k_m) - 1], \tag{15}$$

where $A \equiv n - (1-n)(1-\theta) > 0$ to simplify notations. In (15), the LHS and RHS represent the cost and benefit of holding an additional unit of CBDC, respectively. Knowing (k_b, k_m), (6) gives the equilibrium value of ϕ. We label the Case 1 equilibrium as an unconstrained equilibrium. For any given (i, i_c, i_r), this type of equilibrium exists when χ and δ are big enough so that the constraints are slack. That is,

$$\chi > \frac{\theta[f(k^*) - f(k_m)] + (1-\theta)(k^* - k_m)}{f(k^*)} \equiv \chi_1$$

$$\delta > \frac{(1+i_c)(k^* - k_m)}{(1+i_r)k_m} \equiv \delta_1$$

where k_m solves (15).

In Case 2, k_b is determined in (10). With the binding collateral constraint, we label this type of equilibrium as a collateral constrained equilibrium. From (2) and (14), k_m solves

$$\frac{i - i_c}{1 + i_c} = A[f'(k_m) - 1] + \frac{(n - A)\left[\theta f'(k_m) + 1 - \theta\right]}{(\theta - \chi) f'(k_b) + 1 - \theta}[f'(k_b) - 1]. \tag{16}$$

When the collateral constraint binds, having an additional unit of CBDC helps relax the collateral constraint and increase the loan size, which further lead to a higher k_b. The second term in the RHS of (16) shows this additional benefit of CBDC. We then solve for (k_b, ϕ) from (10) and (6). For this type of equilibrium to exist, it requires $\chi < \chi_1$ and

$$\delta \geq \frac{(1 + i_c)(k_b - k_m)}{(1 + i_r) k_m} \equiv \delta_2(\chi)$$

where (k_m, k_b) solves (10) and (16), and depend on χ. In particular, $\lim_{\chi \to 0} \delta_2(\chi) = 0$ and $\delta_2(\chi_1) = \delta_1$.

In Case 3, (12) includes the bank's reserve d and CBDC $\hat{z}_c$ held by the type-1 entrepreneur. Notice that d comes from deposits by another type-0 entrepreneur. Despite that $d = \hat{z}_c$ in equilibrium, an entrepreneur's choice of $\hat{z}_c$ should not affect k_b through d. It implies that $\partial k_b / \partial \hat{z}_c = 1 + i_c$. Again, (2) and (14) determine k_m from

$$\frac{i - i_c}{1 + i_c} = A[f'(k_m) - 1] + (n - A)[f'(k_b) - 1]. \tag{17}$$

The binding reserve constraint means that an additional unit of CBDC helps relax the reserve constraint and raise the value of k_b. This additional benefit is reflected by the second term in the RHS of (17). Finally, (12) leads to

$$k_b = \left[\frac{\delta(1 + i_r)}{1 + i_c} + 1\right] k_m \tag{18}$$

and ϕ is given by (6). With the binding reserve constraint, we label this type of equilibrium as a reserve constrained equilibrium. The existence of this type of equilibrium requires $\delta < \delta_1$

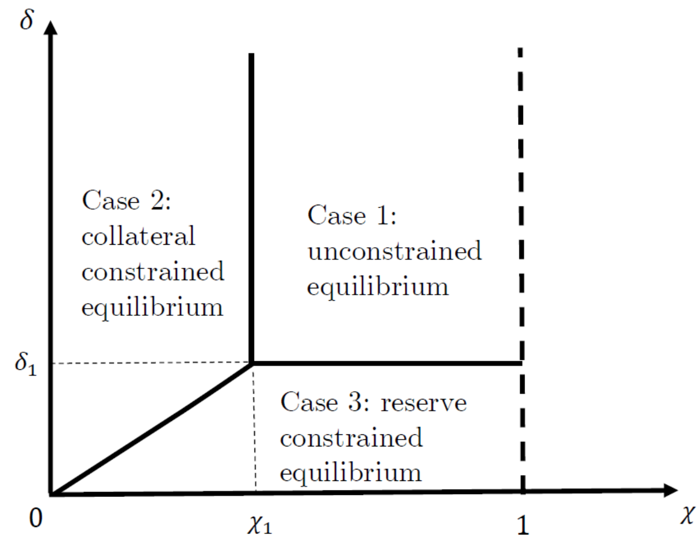

Figure 2: Equilibrium Existence

and

$$\chi \geq \frac{\theta \left[f\left(k_b\right) - f\left(k_m\right)\right] + \left(1-\theta\right)\left(k_b - k_m\right)}{f\left(k_b\right)} \equiv \chi_3\left(\delta\right)$$

where (k_m, k_b) solve (17) and (18), and depend on δ. We can also verify that $\lim_{\delta \to 0} \chi_3\left(\delta\right) = 0$ and $\chi_3\left(\delta_1\right) = \chi_1$.

As discussed above, the collateral constraint and the reserve constraint cannot bind simultaneously for any parameter values. When they both bind, it requires a specific relationship between χ and δ, which is given by the boundary condition $\delta_2\left(\chi\right)$ or $\chi_3\left(\delta\right)$. Moreover, $\delta_2\left(\chi\right)$ or $\chi_3\left(\delta\right)$ are inverse functions. Figure 2 illustrates the existence of the three types of general equilibrium.

4 Policy Analysis

The equilibrium conditions include four policy parameters (i, i_c, i_r, δ). In this section, we use the benchmark model to analyze how policy affects investment, output and banking activities. The interest-bearing aspect of CBDC has raised concerns that CBDC may lead to financial disintermediation. In particular, the interest-bearing CBDC can crowd out bank deposits so that banks' lending activities can be adversely affected. This concern views CBDC and bank deposits as substitutes. In our model, banks can take CBDC as deposits and issue loans. The reserve requirement induces banks to take deposits to make loans. CBDC and deposits become complements. We highlight this new perspective of CBDC because the design of CBDC determines how CBDC and banking activities interact. Through the lens of our model, we examine the effects of the monetary policy parameters (i, i_c, i_r) and the banking policy parameter δ in each type of equilibrium.

Notice that i_r and δ enter into the equilibrium conditions only in the reserve constrained equilibrium. In the other two types of equilibrium, they do not affect the equilibrium allocation as the reserve constraint does not bind. Both i and i_c affect the equilibrium allocation in all types of equilibrium. To be more specific, we focus on the effects of policy on k_b, k_m, aggregate investment $K = (1-n)\,k_b + (2n-1)k_m$, aggregate lending $L = (1-n)\,(k_b - k_m)$, the real deposit rate $r_d = (1+i_d)\,/\,(1+\mu) - 1$ and the real loan rate $r_\ell = \phi/\,(k_b - k_m)$.

In the unconstrained equilibrium, a type-1 entrepreneur invests the optimal amount of capital k^*, which is independent of the policy parameters. Since neither the collateral constraint nor the reserve constraint binds, the economy has a loose credit condition. Banked entrepreneurs can borrow to acquire k^*, and banks are not constrained by the reserve holdings when lending to entrepreneurs. We summarize in Proposition 1 the effects of changing i and i_c. From (15), i and i_c have the opposite effects on the economy because either a higher i or a lower i_c increases the cost of holding CBDC. The Friedman rule can be implemented by setting $i = i_c$. This is consistent with the finding in Nosal and Rocheteau (2017) on interest-bearing money that the implementation of the Friedman rule does not necessarily require deflation. Now consider an increase in i_c. It allows a type-0 entrepreneur to afford more k_m. Since the type-1 entrepreneur borrows $k^* - k_m$ from banks, a higher i_c reduces the amount of bank lending. The simple take-it-or-leave-it offer makes the nominal deposit rate equal to i_c. Therefore, r_d increases, but r_ℓ depends on the banking fee ϕ and the amount of lending $k^* - k_m$. We summarize these results in Proposition 1. Proofs are in the appendix.

Proposition 1 *In an unconstrained equilibrium, a higher i_c leads to a higher k_m, a higher K, a lower L, a higher r_d and a lower r_ℓ. A higher i has the opposite effects.*

The results for the collateral constrained equilibrium are given in Proposition 2. In this equilibrium, only the collateral constraint binds. Hence, this is a situation where banks have enough liquidity, but entrepreneurs are collateral-constrained. From (16), i and i_c again have opposite effects. A higher i_c allows the type-0 entrepreneur to purchase more k_m and the type-1 entrepreneur to have more down payment. Holding more down payment also enables the type-1 entrepreneur to borrow more from banks, so a higher i_c encourages bank lending and investment. The above result is opposite to the result that CBDC can lead to financial disintermediation, where CBDC competes with deposits and crowds out lending. See Andolfatto (2018) and Keister and Sanches (2020) for examples. In our model, CBDC and banking are complements. The deposit contract ensures that the deposit rate is the same as i_c so that type-0 entrepreneurs deposit their idle CBDC.[12] The higher interest rates raise the funding costs for banks, but the deposits allow banks to accumulate reserves to issue loans and make profits. Banks help channel idle liquidity to productive investment.

Proposition 2 *In a collateral constrained equilibrium, a higher i_c leads to a higher k_m, a higher k_b, a higher K, a higher L and a higher r_d, but the effect on r_ℓ is ambiguous. A higher i has the opposite effects.*

In the reserve constrained equilibrium, both i_r and δ can affect investment and banking. In this equilibrium, only the reserve constraint binds. This is a situation where entrepreneurs are not collateral constrained, but banks are constrained by the reserve holdings. It can occur when entrepreneurs face loose credit conditions but the banking policy regulates bank lending tightly.

Proposition 3 *In a reserve constrained equilibrium, a higher i_c leads to a higher k_m and a higher r_d, but is effect on k_b is ambiguous. A higher i leads to a lower k_m, a lower k_b, a lower K, a lower L, a lower r_d and a higher r_ℓ. A higher i_r or δ leads to a lower k_m, a higher k_b, a lower K assuming $f'''(k) > 0$ and a higher L.*

As in the other two cases, a higher i_c leads to a higher k_m because it directly benefits entrepreneurs that rely on internal finance. In the reserve constrained equilibrium, k_b is constrained by the amount of reserves. Reserves held by banks earn interest on reserves, but

[12] The take-it-or-leave-it offer in the deposit market makes $i_d = i_c$. With a more general bargaining share $0 < \gamma < 1$, we would have $i_d > i_c$.

bear the opportunity cost being the interest paid on CBDC. A higher i_c indirectly tightens the reserve constraint. The overall effect of i_c on k_b becomes ambiguous. As a result, its effects on investment and lending are ambiguous. It is worth noticing that the effects of i are not opposite to the effects of i_c in this equilibrium. This is different from the findings in the previous two cases. A higher i lowers the return of CBDC and reduces k_m. Since i does not enter into the reserve constraint directly, k_b also decreases. It follows that both K and L decrease. With no change in i_d, r_d decreases in response to a higher i. For r_ℓ, both ϕ and $k_b - k_m$ decrease, but the decrease in $k_b - k_m$ dominates so that r_ℓ increases.

The reserve constrained equilibrium is an interesting case as i_r plays a non-trivial role. In practice, the interest rate on excess reserves forms a lower bound for the channel system and the floor system operated by modern central banks. Our model suggests that the interest rate on reserves could have an additional role in affecting bank lending and investment in the economy.[13]

It is also worth considering a special case $i_r = i_c$. When reserves earn the same interest rate as CBDC, reserves are less costly. The reserve constraint implies that $k_b = (1 + \delta) k_m$. The investment of type-1 entrepreneurs is proportional to that of type-0 entrepreneurs. It follows that $\partial k_b / \partial i_c > 0$, $\partial K / \partial i_c > 0$, $\partial L / \partial i_c > 0$ and $\partial r_\ell / \partial i_c < 0$. Again, a higher i_c does not lead to financial disintermediation. It allows entrepreneurs to invest more and promotes bank lending. The interest rate on reserves can be adjusted by the central bank to encourage banks to take deposits and issue loans through reducing the funding costs.

Another interesting observation is that a NIR policy is feasible, i.e., $i_c < 0$.[14] In our environment, $i_c < 0$ is not generally preferable because it reduces entrepreneurs' incentives to hold CBDC. This would further lead to less internal finance for unbanked entrepreneurs and less down payment for banked entrepreneurs. There will be less investment if the economy

[13] We discuss the effects of i_c and i_r as separate policy tools. As bank reserves and CBDC belong to central bank high power money, it may also be interesting to further investigate how the interest rates on CBDC and reserves interact in the economy, where the former reflects the cost of liquidity for the public and the latter reflects the cost of liquidity for commercial banks.

[14] Papers related to negative interest rates include He et al. (2008), Rocheteau et al. (2018a), Dong and Wen (2017), and Groot and Haas (2018). He et al. (2008) and Rocheteau et al. (2018a) use New Monetarist models and can generate negative interest rates for assets. Dong and Wen (2017) and Groot and Haas (2018) study the negative interest rate policy which has happened in some advanced economies (such as Japan, Euro Zone, and some European countries), but neither of them is related to CBDC.

is in the unconstrained equilibrium or the collateral constrained equilibrium.

5 Cash and CBDC

CBDC is the only asset in our benchmark model. The complementary role of CBDC and banking makes CBDC essential for banking business. In practice, countries that consider the adoption of CBDC will feature the coexistence of traditional cash and CBDC. Although no country has issued CBDC, it is reasonable to expect that cash and CBDC may coexist for a long time. Even in the extreme case that cash is phased out in the long run, banknote demonetisation in various countries could provide a useful reference for the transition phase. In general, during demonetisation, old series and new series of banknotes coexist in the economy for a limited period. Afterwards old banknotes can no longer circulate, but the public can still go to banks to convert them into new banknotes, within a long time.[15] Given that CBDC bears interest, it is interesting to investigate how CBDC coexists with cash, which has zero interest. Therefore, we use this section to analyze how cash and interest-bearing CBDC can coexist in a model where money and banking serve complementary roles.

The environment remains very similar to the benchmark model, except that entrepreneurs can hold a portfolio of cash and CBDC. We consider the natural case where cash and CBDC have the same value and the same growth rate μ.[16] While entrepreneurs can hold both assets, we consider two extreme scenarios where either cash or CBDC can be accepted by banks as deposits at Stage 1. Specifically, the first scenario is where banks can take only cash as deposits and cash can be turned into reserves. Banks can help entrepreneurs store CBDC, but CBDC cannot be used as reserves. This way of modeling CBDC can be found in Andolfatto (2018), where banks can help individuals store CBDC, but cannot use CBDC to issue loans.

[15] For example, China started the demonetisation of the 4th-series RMB banknotes in 1999, and did not complete the transition to the 5th-series RMB until 2020. The central bank also confirmed cash and DC/EP would coexist for a long time. Switzerland is another good example. In 2019, the Swiss National Bank confirmed it would continue to redeem without time limit the banknotes of the types issued between 1975 and 1993. This also applies to its eighth-series banknotes being recalled as of April 30, 2021.

[16] It might be interesting to consider an endogenous exchange rate between cash and CBDC, but we will focus on the simple and more natural scenario where they have the same value.

In the second scenario, we consider the opposite banking arrangement, where banks accept only CBDC as deposits and CBDC can be turned into reserves. Banks do not accept cash deposits and cash cannot be used by banks as reserves. This type of banking arrangement shares the feature of Internet banks and online banking where banks deal with electronic assets and physical cash is generally not accepted as deposits.

In the following, we briefly outline how adding cash into the model and adjusting the banking arrangement change the value functions and decisions. We also generalize the bargaining power in the deposit market to $\gamma \in (0,1]$. The results from these two models help us understand the potential economic tradeoff between cash and CBDC, and how banking arrangements or regulations can affect these tradeoffs. We begin with the first scenario where only cash is accepted as deposits.

5.1 Cash Only Deposits

With cash as an additional asset, we define $\omega = z + (1+i_c)\, z_c$ as the measure of wealth in the form of cash and CBDC for an entrepreneur at the beginning of Stage 3. In addition to the real balance of CBDC z_c, the entrepreneur has cash z in real terms. For a type-1 entrepreneur, the Stage 3 value function is

$$\begin{aligned} W_1^e(\omega, \ell, k) &= = \max_{x,\hat{z},\hat{z}_c} \{x + \beta \mathbb{E} U^e(\hat{z}, \hat{z}_c)\} \\ \text{st. } x + (1+\mu)\hat{z} + (1+\mu)\hat{z}_c &= \omega - \ell + f(k) + T + \Pi. \end{aligned}$$

Converting to unconstrained optimization, we have the FOCs,

$$1 + \mu = \beta \frac{\partial \mathbb{E} U^e(\hat{z}, \hat{z}_c)}{\partial \hat{z}} = \beta \frac{\partial \mathbb{E} U^e(\hat{z}, \hat{z}_c)}{\partial \hat{z}_c}. \tag{19}$$

The expected value $\mathbb{E}U^e(\hat{z}, \hat{z}_c)$ is

$$\mathbb{E} U^e(\hat{z}, \hat{z}_c) = n U_1^e(\hat{z}, \hat{z}_c) + (1-n)\, U_0^e(\hat{z}, \hat{z}_c). \tag{20}$$

A type-0 entrepreneur has the same FOCs that determine $\hat{z}$ and $\hat{z}_c$, except that $W_0^e(\omega, d)$ has d, the amount of deposits as a state variable. Banks and suppliers have similar value functions as we derive in the benchmark model except that the asset portfolio includes both cash and CBDC.

At Stage 1 in the next period, banks and type-0 entrepreneurs are active in the deposit market, with $U_0^e(\hat{z}, \hat{z}_c) = W_0^e(\omega - d, d)$ and $U_1^e(\hat{z}, \hat{z}_c) = V_1^e(\hat{z}, \hat{z}_c)$, respectively. For banks, $U^b = (1-n) V^b(z_r, d) + nV^b(0,0)$. All deposits are used as reserves $z_r = d$. If a bank does not take deposits, the bank cannot make loans. Again, $V^b(0,0) = W^b(0)$.

At Stage 2 of the loan market, a type-1 entrepreneur has

$$V_1^e(z, \hat{z}) = \frac{1-n}{n} W_1^e(\omega - p_b, \ell, k_b) + \frac{2n-1}{n} W_1^e(\omega - p_m, 0, k_m), \tag{21}$$

where p_b represents the amount of downpayment by banked entrepreneurs and p_m is the amount of payment to purchase capital by unbanked entrepreneurs. For a banked entrepreneur, p_b cannot exceed the total amount of cash and CBDC (including CBDC interest) represented by ω. The entrepreneur can use both internal finance p_b and external finance through a bank loan ℓ to purchase capital. Thus, we again have $\ell = k_b - p_b + \phi$. For an unbanked e, the amount to spend on capital p_m cannot exceed ω. The bank's problem and the supplier's problem remain the same as before.

To solve for the bargaining problems, we begin with the loan contract in the loan market taking the deposit contract (i_d, d) as given. In the loan market, a type-1 entrepreneur's surplus is $f(k_b) - k_b - \phi - [f(k_m) - p_m]$, where $p_m = \hat{z} + (1+i_c)\hat{z}_c = k_m$. Define $\Delta_m \equiv f(k_m) - \hat{z} - (1+i_c)\hat{z}_c$ as the entrepreneur's outside option where cash $\hat{z}$ and CBDC $\hat{z}_c$ are used to purchase capital. Taking $(\hat{z}, \hat{z}_c, i_d, d)$ as given, the Kalai bargaining problem remains the same as in the benchmark model except that the downpayment p_b in the constraints includes cash and CBDC. That is, $p_b \leq \hat{z} + (1+i_c)\hat{z}_c$.We focus on $p_b = \hat{z} + (1+i_c)\hat{z}_c$ because there is no benefit for the type-1 entrepreneur to keep some extra assets given that $i_c \leq i$.

We set up the Lagrangian and let (λ_1, λ_2) be the multipliers associated with the collateral constraint and the reserve constraint, respectively. The FOCs are

$$k_b : \theta \left[f' \left(k_b \right) - 1 \right] = \lambda_1 \left[\left(\theta - \chi \right) f' \left(k_b \right) + 1 - \theta \right] + \lambda_2,$$

$$\lambda_1 : k_b - \hat{z} - \left(1 + i_c \right) \hat{z}_c + \theta \left[f \left(k_b \right) - k_b - \Delta_m \right] - \chi f \left(k_b \right) = 0,$$

$$\lambda_2 : k_b - \delta \left(1 + i_r \right) d - \hat{z} - \left(1 + i_c \right) \hat{z}_c = 0.$$

The solution to the bargaining problem in the loan market again includes three cases, depending on the constraints. Case 1 is the unconstrained equilibrium where neither constraint binds. It gives $k_b = k^*$ and ϕ satisfies (6). Case 2 is the collateral constrained equilibrium with $\lambda_1 > 0$ and $\lambda_2 = 0$. Given $(\hat{z}, \hat{z}_c)$, the bargaining solution (k_b, ϕ, λ_1) satisfy (6), (10) and (11). Case 3 is the reserve constrained equilibrium with $\lambda_1 = 0$ and $\lambda_2 > 0$. The solution (k_b, ϕ, λ_2) is derived from (6), (12) and (13). Notice that an implicit change in these conditions is that both cash and CBDC can be used to purchase capital and $k_m = \hat{z} + (1 + i_c)\, \hat{z}_c$.

In the deposit market, the Nash bargaining problem is

$$\max_{d, i_d} \left[\phi + \left(i_r - i_d \right) d \right]^\gamma \left(i_d d \right)^{1-\gamma} \text{ st. } d \leq \hat{z}.$$

This is essentially the bargaining problem in the benchmark model with $i_c = 0$ because cash has a zero interest rate. When $\gamma = 1$, we have $i_d = 0$. However, for $0 < \gamma < 1$, the choice in the deposit market (i_d, d) could potentially affect ϕ. From the solution of the loan contract, i_d does not directly affect the loan contract and d affects ϕ only in the reserve constrained equilibrium. The FOC with respect to i_d gives $i_d d = (1 - \gamma)(\phi + i_r d)$. It is natural to consider $d = \hat{z}$ because the type-0 entrepreneur should prefer to deposit all cash given that $\gamma < 1$.

5.1.1 General Equilibrium

To complete the description of the general equilibrium, we move back to Stage 3 to determine an entrepreneur's asset choice. The term $\mathbb{E}U^e(\hat{z}, \hat{z}_c)$ now becomes

$$\mathbb{E}U^e(\hat{z}, \hat{z}_c) = (1-n)[f(k_b) - k_b - \phi] + (2n-1)[f(k_m) - k_m] + \hat{z} + (1+i_c)\hat{z}_c$$
$$+ (1-n)(1-\gamma)\left(\phi^d + i_r d\right) + nW_1^e(0,0,0) + (1-n)W_0^e(0,0).$$

We introduce a new notation ϕ^d to represent the banking fee that a depositor's bank earns in the loan market. For a type-0 entrepreneur, it is ϕ^d that matters for their choices of assets. In the extension, ϕ^d depends on d when the reserve constraint binds. Given that $k_m = \hat{z} + (1+i_c)\hat{z}_c$, we have $\partial k_m / \partial \hat{z} = 1$ and $\partial k_m / \partial \hat{z}_c = 1 + i_c$. The expressions of $\partial k_b / \partial \hat{z}$ and $\partial k_b / \partial \hat{z}_c$ depend on the specific type of banking equilibrium.

In the unconstrained equilibrium, $f'(k_b) = 1$ so that k_b does not depend on $\hat{z}$ and $\hat{z}_c$. The banking fee does not depend on the depositor's $(\hat{z}, \hat{z}_c)$. It follows that (19) gives $(\hat{z}, \hat{z}_c)$ solving (15) and

$$i = A[f'(k_m) - 1] + (1-n)(1-\gamma)i_r.$$

Notice that both conditions yield a solution for k_m where $k_m = \hat{z} + (1+i_c)\hat{z}_c$. The coexistence of cash and CBDC requires both conditions to be satisfied, which implies

$$i_r = \frac{i_c(1+i)}{(1-n)(1-\gamma)(1+i_c)} \equiv i_r^*. \tag{22}$$

In this case, CBDC offers a return i_c and cash has a nominal return 0. However, when banks use cash as reserves, cash earns an interest rate i_r. This additional value of cash generates a tradeoff between cash and CBDC. Condition (22) requires a specific relationship between i_c and i_r such that entrepreneurs are indifferent between holding cash and CBDC.[17] The exact portfolio of $(\hat{z}, \hat{z}_c)$ is indeterminate. If (22) is not satisfied, either cash or CBDC is chosen and the other asset is not valued by entrepreneurs.

[17] Technically, the central bank can also adjust i to ensure the coexistence. Since i affects the return of both assets, we focus on i_c and i_r as main policy tools to understand the tradeoff between cash and CBDC.

In the collateral constrained equilibrium, k_b solves (10) and $\partial k_b/\partial \hat{z}$ remains the same as in Case 2 of the benchmark model. Given that the reserve constraint does not bind, the banking fee does not depend on the depositor's cash balance. Then (19) becomes (16) and

$$i = \frac{(n-A)\left[\theta f'(k_m) + 1 - \theta\right]}{(\theta - \chi) f'(k_b) + 1 - \theta}\left[f'(k_b) - 1\right] + A\left[f'(k_m) - 1\right] + (1-n)(1-\gamma)\, i_r.$$

Both conditions give the solution for k_m. Therefore, the coexistence of cash and CBDC boils down to the same condition as (22). We still have an indeterminate portfolio of $(\hat{z}, \hat{z}_c)$. Without this condition, only one asset is valued by entrepreneurs.

In the reserve constrained equilibrium, $k_b = \delta d (1 + i_r) + k_m$. The bank's reserve d comes from another type-0 entrepreneur's deposits. Therefore, the depositor's cash balance can affect its bank's banking fee and $\partial \phi^d / \partial d = \theta \left[f'(k_b) - 1\right] \delta (1 + i_r)$. Despite that $d = \hat{z}$ in equilibrium, a depositor's choice of $\hat{z}$ should not affect k_b through the deposit channel. Therefore, $\partial k_b / \partial \hat{z} = 1$ and $\partial k_b / \partial \hat{z}_c = 1 + i_c$. We can solve for $(\hat{z}, \hat{z}_c)$ from (19), which becomes (17) and

$$i = A\left[f'(k_m) - 1\right] + (n - A)\left[f'(k_b) - 1\right] + (1-n)(1-\gamma)\, i_r + \theta\delta(1-n)(1-\gamma)\left[f'(k_b) - 1\right](1 + i_r) \tag{23}$$

The coexistence of cash and CBDC implies

$$f'(k_b) - 1 = \frac{\frac{i_c(1+i)}{1+i_c} - (1-n)(1-\gamma)\, i_r}{\theta\delta(1-n)(1-\gamma)(1+i_r)}. \tag{24}$$

It gives the solution for k_b. Either (17) or (23) is used to solve for k_m. Knowing (k_m, k_b), $\hat{z}$ is found from the reserve constraint

$$\hat{z} = \frac{k_b - k_m}{\delta(1 + i_r)}. \tag{25}$$

In contrast to the previous two equilibria, the portfolio of $(\hat{z}, \hat{z}_c)$ is now determinate owing to a new tradeoff between cash and CBDC only in the reserve constrained equilibrium. That is, CBDC offers a return i_c, but cash now has an additional benefit by relaxing the reserve constraint. This tradeoff ensures the coexistence of cash and CBDC.

5.1.2 Policy Analysis

In the unconstrained and collateral-constrained equilibria, cash and CBDC can coexist, but the portfolio of $(\hat{z}, \hat{z}_c)$ is indeterminate. When (22) does not hold, either cash or CBDC should be driven out of existence. Suppose i_r is too low to satisfy (22). Then cash is not attractive enough for entrepreneurs to use. Entrepreneurs optimally choose to hold CBDC. However, CBDC cannot be accepted as deposits. If banks do not take deposits, they cannot issue loans. The economy will function as if banks do not exist. All type-1 entrepreneurs will rely on internal finance to purchase capital. Suppose i_r is too high to satisfy (22). Then cash will dominate CBDC and becomes the only asset chosen by entrepreneurs. The economy effectively functions as the benchmark economy with $i_c = 0$ and a generalized Nash bargaining solution. Given that the central bank can adjust both i_r and i_c, it can give up one policy tool (either i_r or i_c) to allow cash and CBDC to coexist by satisfying (22). Therefore, the coexistence of cash and CBDC can be achieved through monetary policy. Proposition 4 summarizes the findings.

Proposition 4 *In the unconstrained and collateral-constrained equilibrium, the coexistence of cash and CBDC requires (22) to hold. Either i_c or i_r cannot be an independent monetary policy tool. The portfolio of $(\hat{z}, \hat{z}_c)$ is indeterminate. If $i_r < i_r^*$, CBDC dominates and the economy collapses to a CBDC-only economy without banks. If $i_r > i_r^*$, cash dominates and the economy functions as the benchmark economy with $i_c = 0$ and $0 < \gamma < 1$.*

The reserve constrained equilibrium is a more interesting case. Cash and CBDC can coexist, and the portfolio of $(\hat{z}, \hat{z}_c)$ is determinate. There is no need for the central bank to sacrifice any monetary policy tool to maintain the coexistence of cash and CBDC. Compared to the previous two equilibria, the tradeoff between cash and CBDC depends on i_r and the extra return on cash through the banking fee. For the equilibrium to exist, i_r cannot be too big for any given i_c. The coexistence of cash and CBDC and the determinate portfolio allow us to investigate how cash and CBDC interact.

Proposition 5 *In the reserve constrained equilibrium, cash and CBDC can coexist when* $i_r \leq i_r^*$. *The portfolio of* $(\hat{z}, \hat{z}_c)$ *is determinate. A higher* i_c *leads to a lower* $\hat{z}$, *a higher* k_m, *a lower* k_b *and a lower* L. *A higher* i_r *or a higher* δ *leads to a higher* k_b, *a lower* k_m *and a higher* L.

When i_c increases, the RHS of (24) increases, which implies that k_b should decrease. From (17), k_m must increase. Given that $k_b - k_m = \delta (1 + i_r) \hat{z}$ and $k_m = \hat{z} + (1 + i_c) \hat{z}_c$, $\hat{z}$ should decrease and $(1 + i_c) \hat{z}_c$ would increase. A higher i_c induces entrepreneurs to hold less cash and the fraction of CBDC in k_m increases. Since only cash serves as reserves, type-0 entrepreneurs deposit less cash and banks issue less loans to banked type-1 entrepreneurs. In this sense, the higher return CBDC crowds out deposits, which reduces the amount of lending in the economy. This result is consistent with the common concern that CBDC might lead to financial disintermediation. The key for this result is that CBDC and banking are no longer complements because banks take cash as deposits. A higher i_c generates a redistribution effect between banked entrepreneurs and unbanked entrepreneurs. An unbanked type-1 entrepreneur purchases capital using his own cash and CBDC. The increase in $(1 + i_c) \hat{z}_c$ dominates the decrease in $\hat{z}$. The unbanked entrepreneur is able to raise k_m in response to a higher i_c. In contrast, despite that a banked entrepreneur's own portfolio allows him to purchase more capital, the reduction in bank lending leads to a lower k_b in response to a higher i_c.

When the reserve constraint binds, both i_r and δ enter into the equilibrium conditions, which enables them to be effective monetary policy tools. A higher i_r or a higher δ directly allows banks to lend more and raises k_b. More lending provided by banks reduces entrepreneurs' incentives to hold assets and k_m decreases.

5.2 CBDC Only Deposits

We now consider the second scenario where only CBDC can be accepted as deposits. At Stage 3, the value functions for a type-1 entrepreneur, a type-0 entrepreneur, a bank and a supplier are the same as the ones discussed in the previous subsection. Moving to Stage 1 in the next period, we have the value function (20) for entrepreneurs where $U_1^e(\hat{z},\hat{z}_c) = V_1^e(\hat{z},\hat{z}_c)$ and $U_0^e(\hat{z},\hat{z}_c) = W_0^e(\omega - d, d)$ for $d \leq \hat{z}_c$. Here d can only take the form of CBDC. For banks, $U^b = (1-n)V^b(z_r, d) + nV^b(0,0)$, where d only comes from CBDC held by a type-0 entrepreneur. At Stage 2, a type-1 entrepreneur has the value function (21). The bank's problem and the supplier's problem remain the same as in the benchmark model.

To solve for the deposit contract and the loan contract, we begin with the bargaining problem in the loan market by taking the deposit contract as given. The bargaining problem that determines the loan contract is the same as in the previous extension. We again consider three cases, and the bargaining solutions for (p_b, k_b, ϕ) are the same as Case 1-3 in the previous extension. In the deposit market, the bargaining problem is the same as (5). Assuming $d = \hat{z}_c$, the FOC for i_d yields

$$i_d d = (1-\gamma)\phi + [\gamma i_c + (1-\gamma) i_r] d. \tag{26}$$

Again, we use the finding that i_d does not directly affect ϕ from the loan contract but d can affect ϕ when the reserve constraint binds.

5.2.1 General Equilibrium

Similarly, after solving the deposit and loan contracts, we can use the solutions to find the asset choice for $(\hat{z},\hat{z}_c)$ at Stage 3. We have

$$\begin{aligned}\mathbb{E}U^e(\hat{z},\hat{z}_c) &= (1-n)[f(k_b) - k_b - \phi] + (2n-1)[f(k_m) - k_m] + \hat{z} + (1+i_c)\hat{z}_c \\ &\quad + (1-n)(1-\gamma)\left[\phi^d + (i_r - i_c)d\right] + nW_1^e(0,0,0) + (1-n)W_0^e(0,0).\end{aligned}$$

In comparison with $\mathbb{E}U^e(\hat{z},\hat{z}_c)$ in the previous extension, $-(1-n)(1-\gamma)i_c d$ appears as an additional term that captures the interest foregone by depositing CBDC. Here ϕ^d is the banking fee earned by the depositor's bank and depends on $\hat{z}_c$ only in the reserve constrained equilibrium. To get $\partial k_b/\partial\hat{z}$ and $\partial k_b/\partial\hat{z}_c$, again we have three cases to consider for the general equilibrium analysis.

In Case 1, the unconstrained equilibrium has $\lambda_1 = 0$, $\lambda_2 = 0$ and $k_b = k^*$. The solution of ϕ satisfies (6). We have $\partial\phi^d/\partial\hat{z}_c = 0$ since the reserve constraint does not bind. Then the FOCs for $\hat{z}$ and $\hat{z}_c$ are

$$
\begin{aligned}
i &= A[f'(k_m) - 1] &(27)\\
\frac{i - i_c}{1 + i_c} &= A[f'(k_m) - 1] + (1 - n)(1 - \gamma)\frac{i_r - i_c}{1 + i_c} &(28)
\end{aligned}
$$

For cash and CBDC to coexist, it requires

$$
i_r = -\frac{[1 + i - (1 - n)(1 - \gamma)]i_c}{(1 - n)(1 - \gamma)} \equiv i_r^{**}. \tag{29}
$$

Since CBDC earns i_c and can be accepted as deposits, this condition implies that i_r must take the opposite sign as i_c to offset the return on CBDC in order for cash to coexist with CBDC.

Case 2 is the collateral constrained equilibrium where $\lambda_1 > 0$ and $\lambda_2 = 0$. From the binding collateral constraint, we can derive $\partial k_b/\partial\hat{z}$ and $\partial k_b/\partial\hat{z}_c$. It follows that the FOCs for $\hat{z}$ and $\hat{z}_c$ are

$$
\begin{aligned}
i &= A[f'(k_m) - 1] + \frac{(n - A)\,[\theta f'(k_m) + 1 - \theta]}{(\theta - \chi)f'(k_b) + 1 - \theta}[f'(k_b) - 1] &(30)\\
\frac{i - i_c}{1 + i_c} &= A[f'(k_m) - 1] + \frac{(n - A)[f'(k_b) - 1]\{1 + \theta[f'(k_m) - 1]\}}{(\theta - \chi)f'(k_b) + 1 - \theta} &(31)\\
&\quad + \frac{(1 - n)(1 - \gamma)(i_r - i_c)}{1 + i_c}. &(32)
\end{aligned}
$$

The coexistence of cash and CBDC requires the same condition as (29). In both Case 1 and Case 2, the portfolio of $(\hat{z}, \hat{z}_c)$ is indeterminate and satisfies $k_m = \hat{z} + (1 + i_c)\hat{z}_c$. A violation of (29) drives one of the assets out of existence.

Lastly, Case 3 is the reserve constrained equilibrium with $\lambda_1 = 0$ and $\lambda_2 > 0$. The binding reserve constraint leads to $\partial k_b/\partial\hat{z} = 1$ and $\partial k_b/\partial\hat{z}_c = 1 + i_c$. Given $k_m = \hat{z} + (1 + i_c)\hat{z}_c$, we have

$$\hat{z}_c = \frac{k_b - k_m}{\delta(1+i_r)}. \tag{33}$$

We then derive the FOCs for $\hat{z}$ and $\hat{z}_c$ as

$$\begin{aligned}
i &= A[f'(k_m) - 1] + (n - A)[f'(k_b) - 1] && (34)\\
\frac{i - i_c}{1 + i_c} &= A[f'(k_m) - 1] + \left[n - A + \frac{\theta\delta(1-n)(1-\gamma)(1+i_r)}{1+i_c}\right][f'(k_b) - 1] && (35)\\
&\quad + \frac{(1-n)(1-\gamma)(i_r - i_c)}{1+i_c}.
\end{aligned}$$

Now the coexistence of cash and CBDC implies

$$f'(k_b) - 1 = -\frac{(1+i)i_c + (1-n)(1-\gamma)(i_r - i_c)}{\theta\delta(1-n)(1-\gamma)(1+i_r)} \tag{36}$$

Using (36), we can solve for k_b. Then k_m and $\hat{z}_c$ are given by (34) and (33). Similar to the findings in the previous extension, the portfolio of $(\hat{z}, \hat{z}_c)$ is determinate because CBDC has a new benefit through relaxing the reserve constraint.

5.2.2 Policy Analysis

In the unconstrained equilibrium and collateral-constrained equilibrium, (29) ensures the coexistence of cash and CBDC. It means that the central bank either imposes a negative i_c accompanied with a positive i_r, or imposes a positive i_c accompanied with a negative i_r. Then entrepreneurs are indifferent in holding either of them. It also means that either i_c or i_r cannot be an independent policy tool.

When $i_r > i_r^{**}$, CBDC dominates and the economy collapses to the benchmark economy (except with generalized Nash bargaining). The effects of changing i_c are similar to the findings in the benchmark model. However, the generalized Nash bargaining makes $i_d > i_c$ and the effects of i_c on r_d are less clear. When $i_r < i_r^{**}$, the economy collapses to a cash-only economy. Since banks accept only CBDC, banks do not function. Proposition 6 summarizes these results.

Proposition 6 *In the unconstrained and collateral-constrained equilibrium, the coexistence of cash and CBDC requires (29) to hold. Either* i_c *or* i_r *cannot be an independent monetary policy tool. The portfolio of* $(\hat{z}, \hat{z}_c)$ *is indeterminate. If* $i_r > i_r^{**}$*, CBDC dominates and the economy functions as the benchmark economy with* $0 < \gamma < 1$*. If* $i_r < i_r^{**}$*, cash dominates and the economy collapses to a cash-only economy without banks.*

As in the previous extension, the reserve constrained equilibrium has more interesting results because cash and CBDC can coexist and the portfolio of $(\hat{z}, \hat{z}_c)$ is determinate. Now CBDC has the additional value in affecting the return on deposits. Again, the existence of the equilibrium requires i_r to be not too big for any given i_c. We summarize our findings in Proposition 7.

Proposition 7 *In a reserve constrained equilibrium, cash and CBDC can coexist when* $i_r \leq i_r^{**}$*. The portfolio of* $(\hat{z}, \hat{z}_c)$ *is determinate. A higher* i_c *leads to a lower* $\hat{z}$*, a higher* $\hat{z}_c$*, a lower* k_m*, a higher* k_b *and a higher* L*. A higher* i_r *or a higher* δ *leads to a higher* k_m*, a lower* k_b *and a lower* L*.*

When i_c increases, the RHS of (36) decreases, which implies k_b should increase. It follows that k_m must decrease from (34). Then (33) implies that $\hat{z}_c$ increases and $k_m = \hat{z} + (1 + i_c)\hat{z}_c$ implies that $\hat{z}$ must decrease. Entrepreneurs are willing to hold more CBDC but less cash. This further leads to more lending by banks. The higher i_c generates a redistribution effect that allows banked entrepreneurs to purchase more k_b and unbanked entrepreneurs to purchase less k_m. In contrast to the finding in the previous extension, the higher i_c promotes banking activities and does not lead to financial disintermediation. The main reason for this result is that CBDC and banking are complements in this extension as banks accept CBDC as deposits. It again highlights that the relationship between CBDC and banking matters for understanding the effect of CBDC on banking.

A practical concern of CBDC is that the interest-bearing CBDC can potentially challenge the existence of cash. In the literature, the coexistence of cash and CBDC typically requires assumptions such as limited participation or segmented markets to prevent some agents from using certain assets. In the absence of such assumptions, we introduce economic tradeoffs between cash and CBDC so that the coexistence of cash and CBDC emerges from these economic tradeoffs. Moreover, the central bank can use proper policy tools to affect these tradeoffs.

6 Discussion and Conclusion

In this paper, we build a benchmark model where CBDC is the only asset in the economy. An important feature of CBDC is that the central bank can pay interest to CBDC through digital accounts. To address the concern that the interest-bearing CBDC may cause financial disintermediation, we show that the relationship between CBDC and banking is critical. A higher CBDC interest rate can encourage investment and bank lending in the benchmark model where CBDC and banking are complements. The interest rate on reserves and the required reserve ratio become effective policy tools in the reserve constrained equilibrium.

We extend the benchmark model by adding cash into the portfolio of entrepreneurs to understand how cash and CBDC interact. We consider two extensions: one in which cash can be accepted as deposits, and the other in which CBDC can be accepted as deposits. In both extensions, the coexistence of cash and CBDC does not require assumptions of limited participation or segmented markets. From the perspective of central bank high power money which includes cash, CBDC and bank reserves, the central bank has the flexibility in adjusting the interest rates of the latter two to ensure the coexistence of cash and CBDC. In the first extension where cash and banking are complements, a higher CBDC interest rate does lead to financial disintermediation whereas in the second extension where CBDC and banking are complements, a higher CBDC interest rate does not lead to financial disintermediation. These results confirm our main finding from the benchmark model that the relationship between CBDC and banking determines whether CBDC can cause financial disintermediation.

CBDC is a new research topic and there are many questions left to explore to help central banks and policy makers understand the implications of issuing CBDC. For example, how should CBDC be issued, through an independent CBDC account system provided by the central bank, or through the current banking infrastructure? In China, the central bank plans to use the two-tier system that relies on the current banking infrastructure. What determines the optimal infrastructure for issuing CBDC for a country? Our paper models CBDC as an interest-bearing money. Another important dimension of CBDC to consider is about privacy: should it be anonymous? At which level of anonymity should it be designed? With the digital account of CBDC, the central bank can potentially access transaction and financial history of all individuals. In contrast, cash transactions are anonymous. Privacy can have benefits and costs. The design of CBDC will have important implications on illegal transactions, tax evasion and the underground economy (Wang, 2021). Additionally, data sharing issues in the era of digital economy (Jones and Tonetti, 2018, Easley et al. 2018) could be important when considering CBDC design. We leave these questions to future research.

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Proof of Proposition 1

For the effects of changing i_c, we have

$$\begin{aligned}
\frac{\partial k_m}{\partial i_c} &= -\frac{1+i}{Af''(k_m)(1+i_c)^2} > 0 \\
\frac{\partial \phi}{\partial i_c} &= -\theta[f'(k_m)-1]\frac{\partial k_m}{\partial i_c} < 0 \\
\frac{\partial L}{\partial i_c} &= -(1-n)\frac{\partial k_m}{\partial i_c} < 0 \\
\frac{\partial r_\ell}{\partial i_c} &= \frac{(k^*-k_m)\frac{\partial \phi}{\partial i_c} + \phi\frac{\partial k_m}{\partial i_c}}{(k^*-k_m)^2}.
\end{aligned}$$

Using the expression of $\partial\phi/\partial i_c$ and $\partial k_m/\partial i_c$, we derive

$$\begin{aligned}
\frac{\partial r_\ell}{\partial i_c} &= \frac{-(k^*-k_m)\,\theta[f'(k_m)-1]+\phi}{(k^*-k_m)^2}\frac{\partial k_m}{\partial i_c} \\
&\simeq \phi-(k^*-k_m)\,\theta\,[f'(k_m)-1] \\
&= f(k^*)-f(k_m)-f'(k_m)(k^*-k_m) < 0
\end{aligned}$$

due to the concavity of the production function.

Proof of Proposition 2

From the collateral constraint,

$$\frac{\partial k_b}{\partial \hat{z}_c} = \frac{(1+i_c)\left[\theta f'(k_m)+(1-\theta)\right]}{(\theta-\chi)f'(k_b)+1-\theta} > 0$$

since (11) implies that the denominator must be positive. For the effects of changing i_c, we use (10) and (16) to derive

$$\begin{aligned}
\frac{\partial k_m}{\partial i_c} &= -\frac{(1+i)[(\theta-\chi)f'(k_b)+1-\theta]}{D(1+i_c)^2} > 0 \\
\frac{\partial k_b}{\partial i_c} &= -\frac{(1+i)[1+\theta(f'(k_m)-1)]}{D(1+i_c)^2} > 0,
\end{aligned}$$

where $D \equiv B_2[1+\theta(f'(k_m)-1)]f''(k_b) + B_1[(\theta-\chi)f'(k_b)+1-\theta]f''(k_m) < 0$ and

$$
\begin{aligned}
B_1 &= \frac{\theta n[f'(k_b)-1] + A[1-\chi f'(k_b)]}{\theta[f'(k_b)-1] + 1 - \chi f'(k_b)} > 0 \\
B_2 &= \frac{(n-A)(1-\chi)[1+\theta(f'(k_m)-1)]}{[(\theta-\chi)f'(k_b)+1-\theta]^2} > 0.
\end{aligned}
$$

For the aggregate lending L,

$$
\begin{aligned}
\frac{\partial L}{\partial i_c} &\simeq \frac{\partial k_b}{\partial i_c} - \frac{\partial k_m}{\partial i_c} \\
&= -\frac{(1+i)\{\chi f'(k_b) + \theta[f'(k_m) - f'(k_b)]}{D(1+i_c)^2} > 0.
\end{aligned}
$$

However, the effects on ϕ and r_ℓ are ambiguous, where

$$
\begin{aligned}
\frac{\partial \phi}{\partial i_c} &= -\frac{\theta(1+i)\{(1-\chi)f'(k_b) - [1-\chi f'(k_b)]f'(k_m)\}}{D(1+i_c)^2} \lesseqgtr 0 \\
\frac{\partial r_\ell}{\partial i_c} &= \frac{\frac{\partial \phi}{\partial i_c}(k_b-k_m) - \phi\frac{\partial(k_b-k_m)}{\partial i_c}}{(k_b-k_m)^2} \lesseqgtr 0.
\end{aligned}
$$

In the case that $\partial\phi/\partial i_c < 0$, we will have $\partial r_\ell/\partial i_c < 0$.

Proof of Proposition 3

From (17) and (18),

$$
\begin{aligned}
\frac{\partial k_m}{\partial i_r} &= -\frac{\delta k_m}{1 + \frac{\delta(1+i_r)}{1+i_c} + \frac{Af''(k_m)}{(n-A)f''(k_b)}} < 0 \\
\frac{\partial k_b}{\partial i_r} &= -\frac{Af''(k_m)}{(n-A)f''(k_b)}\frac{\partial k_m}{\partial i_r} > 0 \\
\frac{\partial K}{\partial i_r} &= (1-n)\frac{\partial k_b}{\partial i_r} + n\frac{\partial k_m}{\partial i_r} \\
&\simeq \frac{A - n^2 f''(k_b) + nA[f''(k_b) - f''(k_m)]}{(n-A)f''(k_b)}
\end{aligned}
$$

If the production function $f(k)$ satisfies $f'''(k) > 0$, then $\partial K/\partial i_r < 0$. Since L depends on $k_b - k_m$, we have $\partial L/\partial i_r > 0$. The real deposit rate does not change as the nominal deposit rate equals i_c. The real loan rate depends on ϕ and $k_b - k_m$, but both ϕ and $k_b - k_m$ increase when i_r increase. Furthermore, the effect of changing i_r on r_ℓ is also ambiguous. Similar to i_r, δ enters only into (18). It has the same effects as i_r.

For the effects of i_c,

$$\frac{\partial k_m}{\partial i_c} = \frac{(n-A)(k_b-k_m)f''(k_b) - (1+i)/(1+i_c)}{(n-A)\left[\delta\left(1+i_r\right)+\left(1+i_c\right)\right]f''\left(k_b\right)+A\left(1+i_c\right)f''\left(k_m\right)} > 0.$$

Given that $i_d = i_c$, $\partial r_d/\partial i_c > 0$. The effects of i_c on (k_b, K, L, r_ℓ) are generally ambiguous.

The effects of i are different from the effects of i_c because i_c enters into (17) and the reserve constraint (18), but i_c only enters into (17). Differentiate (17) and (18) with respect to i,

$$\begin{aligned}
\frac{\partial k_m}{\partial i} &= \frac{\frac{1}{1+i_c}}{Af''(k_m)+(n-A)\,f''\left(k_b\right)\left[\frac{\delta(1+i_r)}{1+i_c}+1\right]} < 0 \\
\frac{\partial k_b}{\partial i} &= \left[\frac{\delta\left(1+i_r\right)}{1+i_c}+1\right]\frac{\partial k_m}{\partial i} < 0.
\end{aligned}$$

It follows that $\partial K/\partial i < 0$. We can rewrite L

$$L = (1-n)\left(k_b-k_m\right) = (1-n)\,\frac{\delta\left(1+i_r\right)}{1+i_c}k_m$$

so that $\partial L/\partial i < 0$. Since $r_d = \left(1+i_d\right)/\left(1+\mu\right)-1 = \beta\left(1+i_c\right)/\left(1+i\right)-1$, $\partial r_d/\partial i < 0$. Given (6), we find

$$\begin{aligned}
\frac{\partial r_\ell}{\partial i} &= \theta\frac{\left[f'\left(k_b\right)\frac{\partial k_b}{\partial i}-f'\left(k_m\right)\frac{\partial k_m}{\partial i}\right]\left(k_b-k_m\right)-\left(\frac{\partial k_b}{\partial i}-\frac{\partial k_m}{\partial i}\right)\left[f\left(k_b\right)-f\left(k_m\right)\right]}{\left(k_b-k_m\right)^2} \\
&\simeq \left\{f'\left(k_b\right)\left(k_b-k_m\right)-\left[f\left(k_b\right)-f\left(k_m\right)\right]\right\}\frac{\partial k_b}{\partial i}-\left\{f'\left(k_m\right)\left(k_b-k_m\right)-\left[f\left(k_b\right)-f\left(k_m\right)\right]\right\}\frac{\partial k_m}{\partial i} \\
&> 0
\end{aligned}$$

because the concavity of the production function implies $f'\left(k_b\right)\left(k_b-k_m\right) < f\left(k_b\right)-f\left(k_m\right)$ and $f'\left(k_m\right)\left(k_b-k_m\right) > f\left(k_b\right)-f\left(k_m\right)$.

Proof for Proposition 5

From (24), a proper solution of k_b requires $k_b < k^*$, which gives rise to the condition

$$\frac{i_c(1+i)}{1+i_c} > (1-n)(1-\gamma)\, i_r$$

and is equivalent to the condition $i_r \leq i_r^*$. We solve for k_b from (24) and k_m from either (17) or (34). When i_c increases, the RHS of (24) increases, which implies that k_b should decrease. Using (17), k_m must increase in response to the higher i_c and the lower k_b. Given that $k_b - k_m = \delta(1+i_r)\hat{z}$ and $k_m = \hat{z} + (1+i_c)\hat{z}_c$, $\hat{z}$ decreases and $(1+i_c)\hat{z}_c$ would increase. In terms of aggregate lending, $L = (1-n)(k_b - k_m)$ must decrease.

When i_r or δ increases, the RHS of (24) decreases, which implies a higher k_b. From (34), the higher k_b implies a lower k_m. It follows that L must increase.

Proof for Proposition 7

For the equilibrium to exist, $k_b \leq k^*$ requires $f'(k_b) - 1 \geq 0$. From (36), we derive $i_r \leq [1-(1+i)/(1-n)(1-\gamma)]i_c = i_r^{**}$. For the effects of changing i_c, we have,

$$\frac{\partial k_b}{\partial i_c} = -\frac{1+i-(1-n)(1-\gamma)}{\theta\delta(1-n)(1-\gamma)(1+i_r)f''(k_b)} > 0.$$

Then (34) implies $\partial k_m/\partial i_c < 0$. It also implies $L = (1-n)(k_b - k_m)$ would increase. Furthermore, we have $\partial \hat{z}_c/\partial i_c > 0$ from (33). Given that $k_m = \hat{z} + (1+i_c)\hat{z}_c$, $\hat{z}$ must decrease in response to a higher i_c.

Similarly, in response to an increase in i_r or δ, the RHS of (36) increases, which leads to a lower k_b. Then (34) implies a higher k_m. It follows that L should decrease.