Liquidity, Monetary Policy and Unemployment*

Mei Dong

University of Melbourne

Sylvia Xiaolin Xiao†

University of Technology, Sydney

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Abstract

We develop a model of money and government bonds to address effects of monetary policy on consumption and unemployment. In the baseline model, money and short-term bonds coexist to facilitate goods trading. We analyze effects of conventional monetary policy when the central bank conducts open market operations by adjusting the relative supply of money and bonds. In general, conventional monetary policy is effective only when the short-term interest rate is positive. We then introduce long-term bonds to examine effects of unconventional monetary policy. We find it is natural to resort to unconventional monetary policy when the short-term interest rate hits the zero lower bound. As in the baseline model, unconventional monetary policy has a redistribution effect on consumption, but its effect on unemployment is ambiguous.

JEL classification: E24, E40, E50

Key words: Liquidity, Zero Lower Bound, Unconventional Monetary Policy, Unemployment

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†Corresponding author: Economics Discipline Group, University of Technology, Sydney, PO Box 123, Broadway NSW 2007, Australia; Email: oceanofsylvia.xiao@gmail.com.

1 Introduction

This paper provides a general framework to analyze the effects of conventional and unconventional monetary policies on macroeconomic performance. Conventional monetary policy in most advanced economies targets short-term interest rates by purchase or sale of short-term government bonds, i.e., open market operations (OMOs). It then *indirectly* affects other market interest rates, including long-term interest rates. The transmission mechanism and effects of such monetary policy have been extensively examined in the literature of monetary economics. The 2008 Global Financial Crisis (GFC) challenges the conduct of conventional monetary policy because the target short-term interest rate in the U.S. has been cut to the zero lower bound. In this case, the Fed is constrained in further lowering the short-term interest rate to stimulate the economy. Similar problems of conventional monetary policy have been observed in Japan and various European countries. In Japan, as early as in 1995, before the outbreak of 1997 Asian Financial Crisis, Bank of Japan cut the short-term interest rate almost to zero, which has lasted till the present. In U.K. and other European countries, since the GFC, central banks have also cut short-term interest rates to the zero lower bound.[1]

After hitting the zero lower bound, central banks of the U.S., Japan, and some European countries all conducted unconventional monetary policy (or so-called Quantitative Easing) by either expanding the holdings of long-term government bonds or directly purchasing public or private assets in financial markets. The goal is to ease conditions of financial markets by *directly* putting downward pressure on long-term interest rates, which will then support economic activities and job creation.

We are interested in the questions of how unconventional monetary policy can affect the returns on money and other assets, and further affect the real economy. To answer these questions, we develop a general equilibrium model which features the coexistence of money and government bonds, and explicit goods and labor markets. For unconventional monetary policy, we focus on bond purchase by the central bank. It is a good approximation, since the ratio of Treasury Bonds holdings is 58.1% in the total securities purchased by the Fed by Jan., 2015, and this ratio is even higher

[1]After the 2008 GFC, the European Central Bank (ECB) has cut the short-term target rate to the zero lower bound, and then to negative, firstly in June, 2014, and furthermore in Sept., 2014. Switzerland has followed a similar path, cutting the short-term target rate from the zero lower bound to negative, firstly in Dec. 2014, and furthermore in Jan., 2015. These may be the only two exceptional cases of imposing negative interest rates in the world since the GFC. However, ECB has decided to start its own Quantitative Easing in March, 2015, since the negative interest rate has not really stimulated credit to the economy, instead, may cause deflation.

in Japan, i.e., 82%. Our modeling of goods market follows the monetary search literature by specifying frictions that make assets (money is one type of assets as well) essential. Our modeling of labor market follows the labor search literature to generate unemployment. Households act as buyers in the goods market and workers in the labor market. Firms are sellers in the goods market and employers in the labor market. Firms' profits from goods market trading directly affect their entry decisions and hence the amount of vacancies in the labor market, which will affect equilibrium unemployment. Moreover, the amount of unemployment directly affects the number of sellers in the goods market. In general, monetary policy affects firms' profits from goods market trading, which will eventually influence unemployment.

In the baseline model, money and short-term government bonds coexist to facilitate goods trading. We model the liquidity difference between money and bonds: short-term bonds are less acceptable and also less pledgeable than money. The central bank can conduct OMOs by adjusting the holdings of short-term government bonds. We find that inflation has a negative impact on unemployment, as previous findings in Berentsen, Menzio and Wright (2011, hereafter BMW). The effect of OMOs is more interesting. Suppose that the central bank purchases short-term bonds to inject money. This will increase demand for short-term bonds, which increases the price but decreases the interest rate of bonds. In the goods market, the lower return of short-term bonds benefits households who do not trade with bonds but hurts households who trade with bonds. We label the opposite effects of OMOs on consumption of different households as a *redistribution effect*. Depending on the fraction of household that trade with bonds, firms' profits may increase or decrease, which further influences the vacancies firms can provide in the labor market. Therefore, unemployment may increase or decrease as a result of the OMOs.

We then extend the model to add long-term government bonds. Long-term bonds differ from short-term bonds in that the former are less pledgeable, and hence offer higher returns than the latter. In this environment, the central bank can conduct unconventional monetary policy by adjusting its holdings of long-term bonds. Our model shows that when the nominal interest rate of short-term bonds is close to zero, without changing the inflation rate, it is essential for the central bank to conduct unconventional monetary policy to influence the economy. Suppose that the central bank purchases long-term bonds to inject money, which resembles Quantitative Easing conducted by central banks in the U.S., and other countries. We find that the redistribution effect is still critical in determining how unconventional monetary policy affects unemployment. However, as in the baseline model, the effect of unconventional monetary policy on unemployment remains ambiguous.

Our paper is related to three lines of literature in monetary economics. The first line of literature integrates monetary search models with labour search models to study monetary policy and unemployment. BMW examine how inflation affects unemployment and show that there is a positive relationship between inflation and unemployment in the long run. They provide a tractable framework where money and unemployment are both modeled with explicit microfoundations. However, as money is the only asset in their model, it is not suitable to address effects of unconventional monetary policy. Other papers built on BMW include Gomis-Porqueras et al. (2013) and Bethune et al. (2014).

The second line of literature involves explicit modeling of assets, liquidity and monetary policy. Along this line, there are many papers, and recent surveys include Williamson and Wright (2010a,b), Nosal and Rocheteau (2011) and Lagos et al. (2014). Here we list a few papers which are more relevant to our paper. Williamson (2012, 2013) builds models with money and government bonds, and also includes banking to address effects of both conventional and unconventional monetary policy. Rocheteau, Wright and Xiao (2014, hereafter RWX) use models with money and government bonds to study OMOs, under various specifications for market structure and for the liquidity of money and bonds. Mahmoudi (2013) builds a model with money, short-term and long-term government bonds, to show that the central bank can change the liquidity and welfare by changing the relative supply of assets with different liquidity characteristics. Since there is no labour market in Williamson (2012, 2013), Mahmoudi (2013) and RWX, they do not consider how unconventional monetary policy affects labor market.

Wen (2013) provides a general equilibrium cash-in-advance (CIA) model featuring government purchases of private debt, to study the efficiency of unconventional monetary policy. The main channel in the model is the trade-off between the quantity and the quality of loans in the private debt market. Herrenbrueck (2013) studies Quantitative Easing and the liquidity channel of monetary policy, through a model with heterogenous households and frictional asset markets. It shows that the central bank purchases of illiquid assets can reduce yields and stimulate investment. Both Herrenbrueck (2013) and Wen (2013) address the effects of unconventional monetary policy on employment, but they model employment through neoclassical production functions, and do not explicit model labor market or job creation. Rocheteau and Rodriguez-lopez (2014) develop a model with an over-the-counter (OTC) market and a Mortensen-Pissarides labor market, to study the joint determination of aggregate liquidity, interest rates and unemployment. For the channel of monetary policy, they focus on the investment channel of firms through the OTC market, while our paper focuses more on the consumption channel of households through the goods market. In addition, they do not explicitly model OMOs as in our paper.

The third line of literature is on OMOs and market segmentation. There are some papers studying OMOs from the perspective of limited participation of agents in the asset market, or transaction cost of transferring money between the assets market and goods market, including Alvarez et al. (2001, 2002, 2009), and Kahn (2006). Most of them use CIA models. And they focus on the short-run effects of OMOs, including the temporary changes in interest rates and economic activity by money injection, the negative relationship between expected inflation and interest rates, or persistent liquidity effects on interest and exchange rates. In some sense, our paper also has market segmentation because bonds are accepted only in some meetings in the goods market. But we model matching between buyers and sellers, and agents go to different types of meetings randomly in the goods market. In addition, those market segmentation papers all focus on conventional monetary policy. Our model can address conventional and unconventional monetary policy.

The New Keynesian literature also has many papers on unconventional monetary policy. Those papers use quite a different approach to address issues related with unconventional monetary policy, including Curdia and Woodford (2011), Gertler and Karadi (2011), Woodford (2012), etc. They either focus on the role of the central bank as financial intermediation to directly provide liquidity in the GFC, or emphasize the central bank using its balance sheet as an instrument of monetary policy. The New Keynesian framework highlights forward guidance as the main channel of unconventional monetary policy (Woodford, 2012). There are mixed empirical evidences on the effectiveness of forward guidance. On one hand, the forward guidance channel depends on the expectation hypothesis of the term structure of interest rates, which has been massively rejected by the data (e.g., Campbell and Shiller, 1991; Bekaert et al., 2001; Kool and Thornton, 2004; Thornton, 2005, 2012; and Sarno et al., 2007). On the other hand, IMF (2013) summarizes empirical work supporting the forward guidance channel of unconventional monetary policy.

The rest of the paper is organized as follows. Section 2 describes the environment. Section 3 introduces the baseline model. We characterize monetary equilibria and analyze effects of conventional monetary policy in Section 4. Section 5 extends the baseline model by adding long-term government bonds. The extended model is then used to address effects of unconventional monetary policy. Section 6 concludes and provides discussion on future research.

2 Environment

Time is discrete and continues forever. As is in BMW, there are three subperiods in each period: in the first subperiod, there is a labor market in the spirit of Mortensen and Pissarides (1994); in the second subperiod, there is a goods market in the spirit of Kiyotaki and Wright (1993); in the last subperiod, there is a frictionless market in the spirt of Arrow-Debreu. We refer to these three markets as MP, KW and AD markets hereafter. There are two types of agents, firms and households, indexed by f and h. The measurement of households is 1, while the measurement of firms is arbitrarily large, but not all firms are active. In addition, there exists a government which is a consolidated fiscal and monetary authority. All government asset transactions take place in AD. Figure 1 shows the timeline of a representative period.

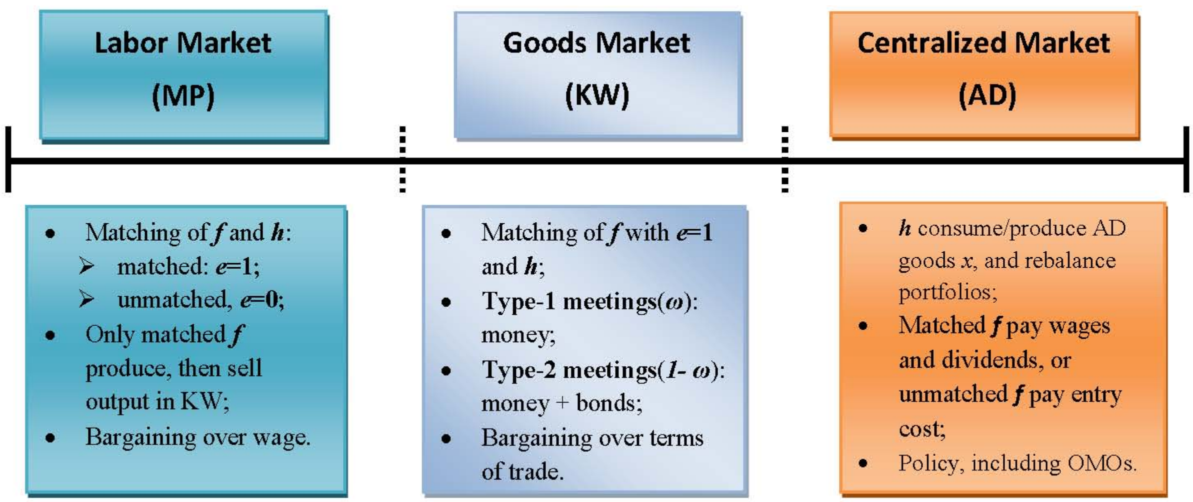

Figure 1: Timeline of a Representative Period

Let e denote employment status: $e = 1$ if a household and a firm are matched, and $e = 0$ if unmatched. In the first subperiod, households and firms match bilaterally to create a job. The matching function $\mathcal{N}(u, v)$, where u is unemployment, and v is vacancies, is constant return to scale (CRS). Once matched, the match produces output y. The wage w is determined by generalized Nash bargaining. The match may break up at an exogenous rate δ. If unmatched, the household is unemployed, and will receive unemployment benefits, κ. Households receive w or κ in the subsequent AD market, instead of the current MP market.

In the second subperiod, all households enter the KW market as buyers of the KW good. The utility from consuming q units of the KW good is $\upsilon(q)$, where $\upsilon(0) = 0$, $\upsilon'(0) = \infty$ and $\upsilon'' < 0 < \upsilon'$. Only firms with output from MP are active in KW as sellers while those unmatched firms in MP skip KW. For active firms, the cost of producing q units of the KW good is $c(q)$ in terms of y, where $c(0) = 0$, $c' > 0$ and $c'' \geq 0$. Buyers and sellers are matched randomly and bilaterally according to the matching technology $\mathcal{M}(B, S)$, which is also CRS, with B and S denoting the measure of buyers and sellers. The terms of trade are determined by bargaining in all meetings.

In the KW market, the roles of households and firms create the double coincidence problem. Since both the KW and AD goods are nonstorable, direct barter is impossible. The lack of commitment implies that pure credit is not viable in the KW market. These frictions make assets necessary as a medium of exchange to facilitate trade. We assume that there are two types of meetings in the KW market. With probability ω, a meeting is type-1, where firms accept only money. With probability $1 - \omega$, the meeting is type-2, where firms accept both money and bonds. Upon matching, an i.i.d. shock determines whether a match is a type-1 meeting or a type-2 meeting.

We then follow Kiyotaki and Moore (1997, 2005, 2012), Nosal and Rocheteau (2013), and RWX to model the liquidity difference (acceptability and pledgeability) among money, and two types of bonds. In the KW market, money is accepted everywhere, but bonds are partially liquid and only accepted in type-2 meetings.[2] In addition, we use a parameter $\gamma_j \in (0, 1]$ to represent the pledgeability of asset j, $j = \{m, s, \ell\}$. It means a fraction γ_j of asset j can be used in KW, either as a payment instrument or collateral.[3] In this paper, we assume $\gamma_m = 1$, since money is accepted everywhere. However, short-term bonds and long-term bonds are partially liquid and less pledgeable so that γ_s and γ_ℓ can be less than 1. In the baseline model, households hold a portfolio of money and short-term bonds, (m, b_s). Later we extend the model to add long-term bonds b_ℓ.

All agents can enter the last subperiod, where a good x is produced and traded in this competitive market. In this market, households receive wage income w (if

[2] One justification is that we need to have at least money and bonds in the model to analyze the effects of OMOs. The existence of type-1 meetings ensures that money is valued, and the existence of type-2 meetings creates a role of government bonds. Moreover, the two types of meetings capture that people have access to different financial resources in the real world. We can endogenize the liquidity difference of money and bonds, by private information (RWX, Lester et al., 2012, and Li et al., 2012), or by mechanism design (Wallace and Zhu, 2007, Nosal and Rocheteau, 2013, and Hu and Rocheteau, 2013, 2014).

[3] See RWX for a more detailed discussion on this.

employed in the previous MP market) or unemployment benefits κ (if unemployed in the previous MP market), dividend income Δ from firms, and government transfers T. For a household, the utility from consuming x units of AD goods is x. If x is negative, it means that households produce x. This linear utility makes households' asset portfolios tractable. The production technology in the AD market is such that 1 unit of labor can produce 1 unit of x. All agents discount between the AD market and the next MP market at a rate β.

The government is active only in the AD market: it issues money and bonds, then buys or sells government bonds to conduct OMOs. The AD prices are ϕ_{mt} for money, ϕ_{st} for short-term bonds, and $\phi_{\ell t}$ for long-term bonds in period t. Let M_t be the money supply in period t. The net growth rate of money supply is π. Both short-term and long-term bonds are nominal. Short-term bonds are one-period bonds, which are issued in period t, and pay 1 unit of money in period $t+1$. Long-term bonds are perpetual bonds (like Consols). Once issued, long-term bonds pay 1 unit of money in every future period.

We will focus on the steady states throughout the paper, so we ignore the time subscript t from now on. To distinguish variables in two sequential periods, for example, (m, b_s) refers to the portfolio of money and short-term bonds upon entering the AD market, while $(\hat{m}, \hat{b}_s)$ is the portfolio carried out of the AD market. We use "+" or "−" in some subscripts to show variables associated with next period, or previous period. We define value functions for the MP, KW and AD markets as U_e^j, V_e^j and W_e^j, where $j \in \{h, f\}$ and $e \in \{0, 1\}$.

Since there are various types of assets in the model, it is useful to distinguish their nominal interest rates. We define the nominal interest rate on an illiquid one-period nominal bond - one that never accepted in the goods market - by the Fisher equation, $1+i = (1+\pi)/\beta$, where $1/\beta = 1+r$ is the real return of the illiquid bond.[4] Let i_s and i_ℓ denote the nominal interest rates of short-term and long-term bonds, respectively. We have

$$1 + i_s = \frac{\phi_m}{\phi_s}, \tag{1}$$

$$1 + i_\ell = \frac{1 + \phi_{\ell+}/\phi_{m+}}{\phi_\ell/\phi_m}, \tag{2}$$

[4]The economic interpretation of i and r is the following: $1+i$ is the amount of money you would need in the next AD to make you indifferent to giving up a dollar today, while $1+r$ is the amount of x you would need in the next AD to make you indifferent to giving up a unit x today. We do not have such an illiquid bond in our model.

where ϕ_{m+} and $\phi_{\ell+}$ are the prices of money and long-term bonds in the next AD market. As in Silveira and Wright (2010), Rocheteau and Rodriguez (2014) and RWX, we define the spreads as the normalized nominal return differences between money and bonds. The spread of short-term bonds s_s and the spread of long-term bonds s_ℓ are

$$s_s = \frac{i - i_s}{1 + i_s}, \tag{3}$$

$$s_\ell = \frac{i - i_\ell}{1 + i_\ell}. \tag{4}$$

3 Baseline Model

In this section, we analyze the value functions for households and firms, and then describe monetary policy. We begin with the value function for households and firms in the current AD market, and then move to the MP and KW markets in the next period.

3.1 Households

A household h entering AD with employment status e and a portfolio of money and short-term bonds (m, b_s), chooses x and the portfolio $(\hat{m}, \hat{b}_s)$ for the next period,

$$W_e^h(m, b_s) = \max_{x, \hat{m}, \hat{b}_s} \left\{ x + (1-e)\chi + \beta U_e^h(\hat{m}, \hat{b}_s) \right\}$$
$$\text{st. } x + \phi_m \hat{m} + \phi_s \hat{b}_s + T = ew + (1-e)\kappa + \Delta + \phi_m m + \phi_m b_s,$$

where χ is the value of leisure. The LHS of the budget constraint is total expenditure, which includes the consumption of x, the value of the portfolio carried to the next period and taxes. The RHS is total income, which includes wage or unemployment benefit, firms' dividend and the value of the household's current portfolio. Notice that the real value of b_s is $\phi_m b_s$ as each unit of bonds pays 1 unit of money at maturity. The real value of the newly acquired bonds is $\phi_s \hat{b}_s$ because the new bonds are issued at the price ϕ_s.

Substituting x from the budget constraint into the value function, we obtain

$$W_e^h(m, b_s) = I_e + \phi_m m + \phi_m b_s + \max_{\hat{m}, \hat{b}_s} \left\{ -\phi_m \hat{m} - \phi_s \hat{b}_s + \beta U_e^h(\hat{m}, \hat{b}_s) \right\}, \tag{5}$$

where $I_e = ew + (1-e)(\kappa + \chi) + \Delta - T$. The envelop conditions give

$$\frac{\partial W_e^h(m, b_s)}{\partial m} = \frac{\partial W_e^h(m, b_s)}{\partial b_s} = \phi_m.$$

As in Lagos and Wright (2005), quasi-linear preferences in the AD market imply that W_e^h is linear in (m, b_s), and the choice of $(\hat{m}, \hat{b}_s)$ is independent of (m, b_s).

For a household in the following MP market,

$$U_1^h(\hat{m}, \hat{b}_s) = \delta V_0^h(\hat{m}, \hat{b}_s) + (1-\delta) V_1^h(\hat{m}, \hat{b}_s), \tag{6}$$

$$U_0^h(\hat{m}, \hat{b}_s) = \lambda_h V_1^h(\hat{m}, \hat{b}_s) + (1-\lambda_h) V_0^h(\hat{m}, \hat{b}_s). \tag{7}$$

where δ is the exogenous job destruction rate and λ_h the endogenous job creation rate. If a household is employed upon entering the MP market, the match may break up with probability δ. Then the household becomes unemployed and $e = 0$. With the rest probability, the household stays employed and carries $e = 1$. Let $\tau = v/u$ be the labor market tightness. We have $\lambda_h = \mathcal{N}(u, v)/u = \mathcal{N}(1, \tau)$. If a household is unemployed upon entering the MP market, the household can find a job with probability λ_h, and the household stays unemployed with the rest probability.

In the subsequent KW market, households become buyers while firms become sellers. Each household is matched at random with a firm. Recall that there are two types of meetings in the KW market. Since households do not know which meetings they will be in, the value function $V_e^h(\hat{m}, \hat{b}_s)$ is a weighted average of $V_e^{h1}(\hat{m}, \hat{b}_s)$ and $V_e^{h2}(\hat{m}, \hat{b}_s)$, where we use superscripts "1" and "2" to denote variables or value functions associated with type-1 and type-2 meetings, respectively. For $e = \{0, 1\}$, we have

$$V_e^h(\hat{m}, \hat{b}_s) = \omega V_e^{h1}(\hat{m}, \hat{b}_s) + (1-\omega) V_e^{h2}(\hat{m}, \hat{b}_s). \tag{8}$$

In type-1 meetings,

$$V_e^{h1}(\hat{m}, \hat{b}_s) = \alpha_h \left[\upsilon(q^1) + W_e^h(\hat{m} - d^1, \hat{b}_s) \right] + (1-\alpha_h) W_e^h(\hat{m}, \hat{b}_s),$$

where $\alpha_h = \mathcal{M}(B, S)/B$ is the household's probability of trade and (q^1, d^1) are the terms of trade in a type-1 meeting. That is, the household uses d^1 units of money to exchange for q^1 units of KW goods. Using the linearity of W_e^h, we can rewrite V_e^{h1} as

$$V_e^{h1}(\hat{m}, \hat{b}_s) = \alpha_h \left[\upsilon(q^1) - \phi_{m+} d^1 \right] + W_{\hat{e}}^h(0, 0) + \phi_{m+}(\hat{m} + \hat{b}_s). \tag{9}$$

In type-2 meetings,

$$V_e^{h2}(\hat{m},\hat{b}_s) = \alpha_h \left[\upsilon(q^2) + W_e^h(\hat{m}-d^2,\hat{b}_s-\mu_s)\right] + (1-\alpha_h)W_e^h(\hat{m},\hat{b}_s),$$

where (q^2, d^2, μ_s) are the terms of trade in a type-2 meeting. The household uses d^2 units of money and μ_s units of short-term bonds to exchange for q^2 units of KW goods. Again, the linearity of W_e^h implies that

$$V_e^{h2}(\hat{m},\hat{b}_s) = \alpha_h[\upsilon(q^2) - \phi_{m+}(d^2+\mu_s)] + W_{\hat{e}}^h(0,0) + \phi_{m+}(\hat{m}+\hat{b}_s). \tag{10}$$

Substituting V_e^{h1} and V_e^{h2} from (9) and (10) into (8), we have

$$V_e^h(\hat{m},\hat{b}_s) = \alpha_h S_h + W_{\hat{e}}^h(0,0) + \phi_{m+}(\hat{m}+\hat{b}_s). \tag{11}$$

We define $S_h = \omega[\upsilon(q^1) - \phi_{m+}d^1] + (1-\omega)[\upsilon(q^2) - \phi_{m+}(d^2+\mu_s)]$ as the household's expected surplus from trading in the KW market. All households participate in the KW market, so $B = 1$; only firms with $e = 1$ enter the KW market, so $S = 1-u$, where u denotes the fraction of unemployed households. Therefore, we have $\alpha_h = \mathcal{M}(1, 1-u)$. Furthermore, we can define $\alpha_h \equiv \alpha_h(u)$. This establishes a connection between the labor and goods market. Higher unemployment in the labor market means less active firms in the goods market, thus the trading probability is lower for households in the goods market, i.e., $\alpha_h'(u) < 0$.

We substitute (11) into (6) and (7),

$$\begin{aligned}
U_1^h(\hat{m},\hat{b}_s) &= \alpha_h S_h + \phi_{m+}(\hat{m}+\hat{b}_s) + \delta W_0^h(0,0) + (1-\delta)W_1^h(0,0), \\
U_0^h(\hat{m},\hat{b}_s) &= \alpha_h S_h + \phi_{m+}(\hat{m}+\hat{b}_s) + \lambda_h W_1^h(0,0) + (1-\lambda_h)W_0^h(0,0),
\end{aligned}$$

or,

$$U_e^h(\hat{m},\hat{b}_s) = \alpha_h S_h + \phi_{m+}(\hat{m}+\hat{b}_s) + \mathbb{E}W_{\hat{e}}^h(0,0). \tag{12}$$

where $\mathbb{E}W_{\hat{e}}^h(0,0)$ is the expectation with respect to next period's employment status. It is clear that $\partial U_e^h/\partial\hat{m}$ and $\partial U_e^h/\partial\hat{b}_s$ do not depend on the employment status. We then substitute (12) into the maximization problem of (5),

$$\begin{aligned}
W_e^h(m,b_s) = I_e &+ \phi_m m + \phi_m b_s + +\beta\mathbb{E}W_{\hat{e}}^h(0,0) + \\
&\max_{\hat{m},\hat{b}_s}\left\{-\phi_m\hat{m} - \phi_s\hat{b}_s + \beta\left[\alpha_h S_h + \phi_{m+}(\hat{m}+\hat{b}_s)\right]\right\}.
\end{aligned} \tag{13}$$

From (13), the choice of $(\hat{m},\ \hat{b}_s)$ is independent of e and (m, b_s). Hence, every household takes the same portfolio of money and bonds out of the AD market.

3.2 Firms

In the AD market, the portfolio decisions by firms are trivial. Firms would not carry any money or bonds out of the AD market since they would not use money or bonds in the subsequent MP or KW markets. For a matched firm with inventory x, money balances m and short-term bonds b_s, its value function in the AD market is

$$W_1^f(x, m, b_s) = x + \phi_m m + \phi_m b_s - w + \beta U_1^f. \tag{14}$$

The envelope conditions imply that

$$\frac{\partial W_1^f(x, m, b_s)}{\partial x} = 1 \text{ and } \frac{\partial W_1^f(x, m, b_s)}{\partial m} = \frac{\partial W_1^f(x, m, b_s)}{\partial b_s} = \phi_m.$$

As firms do not carry any money or bonds into the MP and KW markets, we omit the state variables in U_e^f and V_e^f without loss of generality.

Depending on a firm's employment status, the firm's value function in the following MP market is

$$\begin{aligned} U_1^f &= \delta V_0^f + (1-\delta) V_1^f, \\ U_0^f &= \lambda_f V_1^f + (1-\lambda_f) V_0^f, \end{aligned} \tag{15}$$

where $\lambda_f = \mathcal{N}(u, v)/v = \mathcal{N}(1, \tau)/\tau$, is the endogenous job filling rate. For a matched firm upon entering the MP market, the match may break with probability δ so that the firm becomes vacant. With probability $1-\delta$, the firm stays matched. For a vacant firm upon entering the MP market, the firm can find a household with probability λ_f. With the rest probability, the firm stays vacant. As is mentioned before, only matched firms in MP become *active* in the subsequent KW, while unmatched firms skip KW.

In the KW market, a firm may meet a household in a type-1 or type-2 meetings. Its value function is,

$$V_1^f = \omega V_1^{f1} + (1-\omega) V_1^{f2}, \tag{16}$$

where

$$V_1^{f1} = \alpha_f W_1^{f1}[y - c(q^1), \phi_{m+}d^1, 0] + (1-\alpha_f)W_1^{f1}(y, 0, 0)\,, \tag{17}$$

$$V_1^{f2} = \alpha_f W_1^{f2}[y - c(q^2), \phi_{m+}d^2, \phi_{m+}\mu_s] + (1-\alpha_f)W_1^{f2}(y, 0, 0)\,. \tag{18}$$

Here $\alpha_f = \mathcal{M}(B,S)/S$ is the firm's probability of trade. Similar to α_h, we have $\alpha_f = \mathcal{M}(1, 1-u)/(1-u) \equiv \alpha_f(u)$. Higher unemployment in MP means less active firms in KW, thus the trading probability is higher for firms in KW, i.e., $\alpha_f'(u) > 0$. As in BMW, for $j = \{1, 2\}$, it costs a firm $c\,(q^j)$ units of goods produced in MP to produce q^j units of KW goods. The firm can carry the rest $y - c(q^j)$ as inventory to the subsequent AD market. Using the linearity of W_e^h in (x, m, b_s), we can rewrite (17) and (18), and then substitute them into (16) as,

$$V_1^f = y - w + \alpha_f S_f + \beta U_1^f, \tag{19}$$

where $S_f = \omega\,[\phi_m d^1 - c(q^1)] + (1-\omega)[\phi_m d^2 + \phi_m \mu_s - c(q^2)]$. Similar to S_h, S_f is the firm's expected surplus from trading in the KW market. Using (19) and (15), we can express V_1^f as

$$V_1^f = \frac{y - w + \alpha_f S_f}{1 - \beta(1-\delta)}. \tag{20}$$

The free entry condition in the AD market implies that firms with $e = 0$ can choose to enter the AD market freely by paying a cost k. Thus we have

$$W_0^f = \max\left\{0, -k + \beta\lambda_f V_1^f + \beta(1-\lambda_f)V_0^f\right\},$$

where $V_0^f = W_0^f = 0$ in equilibrium. It follows that $k = \beta\lambda_f V_1^f$. All of this is standard, as in Mortensen and Pissarides (1994). Combined with (20), we have,

$$k = \frac{\beta\lambda_f(y - w + \alpha_f S_f)}{1 - \beta(1-\delta)}. \tag{21}$$

Recall that firms pay out profits as dividends in the AD market. The overall profit by all firms is $(1-u)(y - w + \alpha_f S_f) - vk$. For a household who owns shares of all firms, the dividend income is $\Delta = (1-u)(y - w + \alpha_f S_f) - vk$.

3.3 Government

The government is a consolidated fiscal and monetary authority. We focus on its role as the central bank and treat the fiscal policy as passive. Consider a representative period. Suppose that the government holds a balanced budget in every period. The government budget constraint is

$$\phi_m \left(M - M_{-}\right) + \phi_s B_s + T = \phi_m B_{s-} + u\kappa, \tag{22}$$

Here M and B_s are money supply and short-term bonds supply in the current period, while M_{-} and B_{s-} are those from previous period. The LHS of (22) shows the government's total revenue, which includes the real value of newly issued money and bonds plus the tax revenue. The RHS of (22) represents the government's total expenditure, which includes the redemption of the previously issued government bonds and the unemployment benefit paid to unemployed households.

In terms of monetary policy, the government can either adjust the growth rate of money supply or the relative supply of money and bonds. In the steady state, adjusting π is equivalent to adjusting i, from the Fisher equation. We use σ_s to denote the ratio of short-term government bonds to money. The government commits to monetary policy where money supply grows at $1 + \pi$, and the ratio of short-term bonds to money is a constant σ_s. That is,

$$\frac{M}{M_{-}} = 1 + \pi, \tag{23}$$

$$\frac{B_s}{M} = \sigma_s. \tag{24}$$

Given money and bonds grow at the same rate π, adjusting σ_s is a representation of OMOs by the central bank, . For example, decreasing σ_s means that the central bank buys bonds, thus decreasing the bond holdings by the public, to inject money. Conversely, increasing σ_s means that the central bank sells bonds to withdraw money from the public. It is consistent with the OMOs setting in RWX. Williamson (2012) defines OMOs in a similar way.

To summarize, the central bank has two monetary policy tools: i and σ_s. OMOs by adjusting σ_s generally affect the short-term interest rate of bonds, i_s. In monetary equilibrium, the no-arbitrage condition implies that i_s must not be lower than the nominal return of money (i.e., 0), and quasilinear preferences imply that i_s cannot exceed i. Therefore,

$$0 \leq i_s \leq i. \tag{25}$$

4 Equilibrium

The terms of trade in three markets are determined as follows: agents are price takers in the AD market, and bargain over the terms of trade in the MP and KW markets. In this section, we solve for equilibrium conditions in all markets and define stationary monetary equilibria. Then we use the model to analyze effects of monetary policy.

4.1 Goods Market Equilibrium

When a firm and a household meet in the KW market, the terms of trade are determined by bargaining in all meetings. A generic way to define the bargaining solution is that a household pays $g(q^j)$ to purchase q^j units of the KW good, where $g(\cdot)$ depends on the specific bargaining protocol. For example, the bargaining protocol could be Kalai bargaining (Kalai, 1977) or generalized Nash bargaining. Let θ be the household's bargaining power. Consider Kalai bargaining: a household and a firm in type j meetings ($j \in \{1, 2\}$) bargain to

$$\max_{q^j, d^j, \mu_s^j} \left[\upsilon\left(q^j\right) - \phi_{m+}(d^j + \mu_s \cdot \mathbf{I}^j) \right]$$
$$\text{st. } \upsilon\left(q^j\right) - \phi_{m+}(d^j + \mu_s \cdot \mathbf{I}^j) = \theta \left[\upsilon\left(q^j\right) - c\left(q^j\right) \right]$$
$$d^j \leq \hat{m} \text{ and } \mu_s \leq \gamma_s \hat{b}_s.$$

The indicator $\mathbf{I}^j$ is such that $\mathbf{I}^1 = 0$ and $\mathbf{I}^2 = 1$. The parameter γ_s reflects that only a fraction of short-term bonds can be accepted during exchange. The solution to Kalai bargaining is the following. In case that $d^j = \hat{m}$ and $\mu_s = \gamma_s \hat{b}_s$, we have

$$g(q^j) = \phi_{m+}(\hat{m} + \gamma_s \hat{b}_s \cdot \mathbf{I}^j) = (1-\theta)\upsilon(q^j) + \theta c(q^j).$$

In case that either $d^j < \hat{m}$ or $\mu_s < \gamma_s \hat{b}_s$, we have

$$q^j = q^* \text{ where } q^* \text{ solves } \upsilon'(q) = c'(q),$$
$$\text{and } g(q^j) = \phi_{m+}(d^j + \gamma_s \mu_s \cdot \mathbf{I}^j) = (1-\theta)\upsilon(q^*) + \theta c(q^*).$$

In the generalized Nash bargaining, one can show that

$$g(q^j) = \frac{\theta c(q^j)\upsilon'(q^j) + (1-\theta)\upsilon(q^j)c'(q^j)}{\theta \upsilon'(q^j) + (1-\theta)c'(q^j)}.$$

For now, we use the general bargaining solution where the payment for exchanging

q^j units of the KW good is $g(q^j)$. Note that another implicit constraint associated with the bargaining problem is $c(q^j) \leq y$. It means that a firm's supply of q^j is restricted by the output y produced in the labor market. Following BMW, throughout this paper, we assume that y is big enough so that this constraint is never binding. Lemma 1 summarizes the bargaining solution.

Lemma 1 *In type-1 meetings, the bargaining solution is*

$$d^1 = \hat{m},\ q^1 = g^{-1}(\phi_{m+}\hat{m}); \tag{26}$$

and in type-2 meetings, the bargaining solution is

$$d^2 = \hat{m},\ q^2 = \begin{cases} g^{-1}(\phi_{m+}\hat{m} + \phi_{m+}\gamma_s \hat{b}_s), & \text{if } \mu_s = \gamma_s \hat{b}_s \\ q^*, & \text{if } \mu_s < \gamma_s \hat{b}_s \end{cases} \tag{27}$$

Proof 1 *As in Lagos and Wright (2005), it is costly for households to hold money when $i > 0$. Households generally spend all the money carried to the KW market. Hence, the cash constraint is always binding, i.e., $d^1 = d^2 = \hat{m}$. In type-2 meetings, whether $\mu_s \leq \gamma_s \hat{b}_s$ is binding depends on the return of bonds. When $\mu_s = \gamma_s b_s$, q^2 is solved from $g(q^2) = \phi_{m+}(\hat{m} + \gamma_s \hat{b}_s)$. When $\mu_s < \gamma_s \hat{b}_s$, $q^2 = q^*$.*

Given the bargaining solution in Lemma 1, we move back to the AD market and solve for interior solutions from the FOCs of $(\hat{m}, \hat{b}_s)$ in (13),

$$i = \alpha_h(u)\left[\omega\lambda(q^1) + (1-\omega)\lambda(q^2)\right], \tag{28}$$

$$s_s = \alpha_h(u)(1-\omega)\gamma_s\lambda(q^2). \tag{29}$$

We use $\lambda(q^j) = v'(q^j)/g'(q^j) - 1$ to denote the liquidity premium in a type-j meeting, as in RWX. This is also the Lagrangian multiplier on the bargaining constraints of type-j meetings. In (28), the LHS i is the marginal cost of spending 1 more unit of money for households, while the RHS is the expected marginal benefit of spending 1 more unit of money, from two types of meetings. Similarly, in (29), the LHS s_s is the marginal cost of spending 1 more unit of short-term bonds for households, while the RHS is the marginal benefit of spending 1 more unit of short-term bonds, only from type-2 meetings. Kalai bargaining implies that $\lambda'(\cdot) < 0$. Lagos and Wright (2005) discuss the condition that ensures $\lambda'(\cdot) < 0$ under generalized Nash bargaining.

When $\mu_s = \gamma_s \hat{b}_s$ in type-2 meetings, (26)-(29) determine the equilibrium $(\hat{m}, \hat{b}_s, q^1, q^2, i_s)$, given u, and the asset market clearing condition (24) holds. Particularly, (28) and

(29) show the linkage between labor market and goods market: more unemployment reduces the number of firms entering into KW and hence reduces the matching probability for households, which will further affect equilibrium (q^1, q^2). In addition to this case, we will provide a full description of monetary equilibrium in Section 4.3.

4.2 Labor Market Equilibrium

In the MP market, wage is determined by generalized Nash bargaining. Let η be the bargaining power of a firm. Similar to Mortensen and Pissarides (1994), we get

$$w = \frac{\eta[1-\beta(1-\delta)](b+\chi) + (1-\eta)[1-\beta(1-\delta-\lambda_h)]\,(y+\alpha_f S_f)}{1-\beta(1-\delta)+(1-\eta)\beta\lambda_h}. \tag{30}$$

Substituting (30) into (21), the free entry condition becomes

$$k = \frac{\lambda_f \eta\,(y-\kappa-\chi+\alpha_f S_f)}{r+\delta+(1-\eta)\lambda_h}. \tag{31}$$

In the steady state, the flow condition in the labor market implies that $(1-u)\delta = \mathcal{N}(u, v)$. This implicitly defines $v = v(u)$. Now we have $\lambda_f = \mathcal{N}[u, v(u)]/v(u)$ and $\lambda_h = \mathcal{N}[u, v(u)]/u$. It follows that we can rewrite $\lambda_f \equiv \lambda_f(u)$, and $\lambda_h \equiv \lambda_h(u)$. Higher unemployment decreases the labor market tightness τ, which will increase the matching probability of firms in MP but decrease that of households. Therefore, we have $\lambda_f'(u) > 0$ and $\lambda_h'(u) < 0$. Here with a general bargaining power parameter, the Hosios (1990) condition does not hold. Recall that $\alpha_f = \alpha_f(u)$. We then update (31) as

$$k = \frac{\eta\lambda_f(u)\,[y-\kappa-\chi+\alpha_f(u)S_f]}{r+\delta+(1-\eta)\lambda_h(u)}. \tag{32}$$

This free entry condition determines u, given (q^1, q^2). Compared to the free entry condition in BMW, the firm's expected trading surplus S_f in (32) is the expected surplus from both type-1 and type-2 meetings.

We can rewrite (32) as

$$H(u) = \omega\left[g(q^1)-c(q^1)\right] + (1-\omega)\left[g(q^2)-c(q^2)\right], \tag{33}$$

where $H(u) = \{k\,[\,r+\delta+(1-\eta)\lambda_h(u)]\,/\,[\,\eta\lambda_f(u)] - (y-\kappa-\chi)\}\,/\alpha_f(u)$. We have $H'(u) < 0$, since $\lambda_h'(u) < 0$, $\lambda_f'(u) > 0$ and $\alpha_f'(u) > 0$. The RHS of (33) is the firm's expected gain in KW, i.e., S_f, given the bargaining solution. Intuitively, when both q^1 and q^2 are higher, firms' profits and hence the benefit from opening a vacancy are higher, so unemployment will decrease. This channel is the same as in BMW. In our model, there are two types of meetings in the KW market. If q^1 and q^2 change in opposite directions, their effect on unemployment becomes ambiguous. We leave more discussion in Section 4.3.

4.3 Equilibrium Allocation

After solving the equilibrium in each market, we can combine the equilibrium conditions to define the general equilibrium allocation. As mentioned before, we focus on stationary equilibrium.

Definition 1 *Given monetary policy parameters* (i, σ_s)*, a stationary monetary equilibrium is a list of* $(\hat{m}, \hat{b}_s, q^1, q^2, i_s, u)$ *such that (i) given* u*,* $(\hat{m}, \hat{b}_s)$ *solves (13),* (q^1, q^2) *satisfies (26) and (27), and* i_s *satisfies (1);(ii) given* $(\hat{m}, \hat{b}_s, q^1, q^2, i_s)$*,* u *satisfies (32); and (iii) the asset market clears, (23)-(24) hold, and*

$$\hat{b}_s = \sigma_s \hat{m}. \tag{34}$$

Proposition 1 *Stationary monetary equilibrium exists if* $k < \eta(y - \kappa - \chi)/(r + \delta)$ *and* $\pi \geq \beta - 1$.

Similar to the findings in BMW, there always exists a non-monetary equilibrium for any k. When monetary equilibrium exists, it may not be unique. From (25), there can be four types of monetary equilibrium depending on the value of i_s. When $0 = i_s = i$, it is the Friedman Rule Equilibrium where $q^1 = q^2 = q^*$. When $0 = i_s < i$, households do not have any incentive to carry bonds, and the economy is in a liquidity trap. OMOs become irrelevant. When $0 < i_s = i$, it is costless for households to hold bonds. Households may accumulate an infinite amount of bonds, and $q^2 = q^*$ in type-2 meetings. Again OMOs are irrelevant. Lastly, when $0 < i_s < i$, households carry both money and bonds. This is the only case where OMOs matter. We analyze each type of equilibrium in the following.

4.3.1 Friedman Rule Equilibrium

Lastly, when $0 = i_s = i$, both money and bonds have zero nominal returns. Since $\pi = \beta - 1$, the real return of bonds equals with the inverse of time preference. In this case, we are in a Friedman rule equilibrium. The equilibrium allocation is $q^1 = q^2 = q^*$ and u is solved from (32). The Friedman rule equilibrium exists if and only if $\pi = \beta - 1$.

4.3.2 Liquidity Trap Equilibrium

When $0 = i_s < i$, bonds have the same nominal return as money. There is no room to further lower the short-term interest rate, thus we label it as the liquidity trap equilibrium. Since bonds are accepted only in type-2 meetings, households hold only money. It follows that $q^1 = q^2 = q$. To simplify notations, from now on we denote the real balances of money as $\hat{z} = \phi_{m+}\hat{m}$, except stated otherwise. The equilibrium $(\hat{z}, q, u)$ satisfy

$$\begin{aligned} g(q) &= \hat{z}, \\ i &= \alpha_h(u)\lambda(q), \\ H(u) &= g(q) - c(q). \end{aligned}$$

This economy becomes a pure monetary economy, exactly as in BMW. In this liquidity trap equilibrium, OMOs are irrelevant. The liquidity trap equilibrium exists if and only if $(i, \sigma_s) \in$

$$\{(i, \sigma_s) : 0 = i_s < i, \sigma_s = 0\}.$$

4.3.3 Equilibrium with Plentiful Bonds

When $0 < i_s = i$, it is costless to hold bonds since the real return of bonds is just the inverse of time preference. Therefore, households may choose to hold an infinite amount of bonds. This is also corresponding to the constraint in type-2 meetings is not binding, i.e., $\mu_s < \gamma_s \hat{b}_s$. We label this as equilibrium with plentiful bonds. In this case, the equilibrium $(\hat{z}, q^1, q^2, i_s, u)$ satisfy

$$\begin{aligned} g(q^1) &= \hat{z}, \\ q^2 &= q^*, \\ i &= \alpha_h(u)\omega\lambda(q^1), \\ H(u) &= \omega \left[g(q^1) - c(q^1) \right] + (1-\omega) \left[g(q^*) - c(q^*) \right]. \end{aligned}$$

Notice that $i_s = i$, then $s_s = 0$. In addition, since households get the first best q^* in type-2 meetings, we have $\lambda(q^2) = 0$. Again OMOs are irrelevant. Define the solution to q^1 as q^{1p} (the superscript "p" refers to the case of plentiful bonds). The equilibrium with plentiful bonds exists if and only if $(i, \sigma_s) \in$

$$\left\{(i, \sigma_s) : 0 < i_s = i,\ \sigma_s \geq \sigma^*\right\}.$$

where $\sigma^* = [g(q^*) - g(q^{1p})]/[\gamma_s g(q^{1p})]$.

4.3.4 Equilibrium with Scarce Bonds

When $0 < i_s < i$, the nominal interest rate of bonds is positive but less than i. In this case, the return on bonds is not so high and households hold a finite amount of bonds. This is also corresponding to the constraint in type-2 meetings is binding, i.e., $\mu_s = \gamma_s \hat{b}_s$. We label this as equilibrium with scarce bonds. This is the most interesting case in the baseline model. As is discussed later, it is the only case OMOs matter.

The equilibrium $(\hat{z}, q^1, q^2, i_s, u)$ satisfy (28), (29), (33) and

$$g(q^1) = \hat{z}, \tag{35}$$

$$g(q^2) = (1 + \gamma_s \sigma_s)\hat{z}. \tag{36}$$

Notice that i_s is determined by s_s in (3) while s_s is solved in (29). Furthermore, from (35)-(36), we can derive

$$\frac{\partial q^1}{\partial \hat{z}} = \frac{1}{g'(q^1)} > 0,\ \frac{\partial q^2}{\partial \hat{z}} = \frac{1 + \gamma_s \sigma_s}{g'(q^2)} > 0,\ \frac{\partial q^2}{\partial \sigma_s} = \frac{\gamma_s \hat{z}}{g'(q^2)} > 0. \tag{37}$$

where $g'(q^j) > 0$ means households need to pay more to buy more q^j. A necessary condition for the equilibrium with scarce bonds to exist is $(i, \sigma_s) \in$

$$\left\{(i, \sigma_s) : 0 < i_s < i,\ 0 < \sigma_s < \sigma^*\right\}.$$

We can analyze the effects of monetary policy (i, σ_s) by taking full derivation against (28)-(29) and (35)-(36). For the effects of changing i, we have

$$\frac{\partial \hat{z}}{\partial i} = -\frac{g_1' g_2' H'(u)}{D} \simeq D, \tag{38}$$

$$\frac{\partial q^1}{\partial i} = -\frac{g_2' H'(u)}{D} \simeq D, \tag{39}$$

$$\frac{\partial q^2}{\partial i} = -\frac{(1 + \gamma_s \sigma_s)\, g_1' H'(u)}{D} \simeq D, \tag{40}$$

$$\frac{\partial u}{\partial i} = -\frac{\omega g_2' \left(g_1' - c_1'\right) + (1 - \omega)\left(1 + \gamma_s \sigma_s\right) g_1' \left(g_2' - c_2'\right)}{D} \simeq -D, \tag{41}$$

where $D = -\alpha_h'(u)\left[\omega \lambda_1 + (1 - \omega)\lambda_2\right]\Omega - \alpha_h(u) H'(u)\left[\omega g_2' \lambda_1' + (1 - \omega)\left(1 + \gamma_s \sigma_s\right) g_1' \lambda_2'\right]$ and $\Omega = \left[\omega g_2'(g_1' - c_1') + (1 - \omega)\left(1 + \gamma_s \sigma_s\right) g_1'(g_2' - c_2')\right] > 0$. The first item of D is positive, while the second is negative. Therefore, it is possible that $D < 0$, or $D > 0$. We use "$\simeq$" to show that two expressions have the same sign. From (38)-(41), the effects of changing i are directly determined by the sign of D.

For the effects of OMOs, it is reflected by changing the parameter σ_s. We have

$$\frac{\partial \hat{z}}{\partial \sigma_s} = \frac{(1-\omega)\gamma_s \hat{z} g_1' \left[\alpha_h(u)\lambda_2' H'(u) + i\alpha_h'(u)(g_2' - c_2')/\alpha_h(u)\right]}{D}, \tag{42}$$

$$\frac{\partial q^1}{\partial \sigma_s} = \frac{(1-\omega)\gamma_s \hat{z} \left[\alpha_h(u)\lambda_2' H'(u) + i\alpha_h'(u)(g_2' - c_2')/\alpha_h(u)\right]}{D}, \tag{43}$$

$$\frac{\partial q^2}{\partial \sigma_s} = -\frac{\omega\gamma_s \hat{z} \left[\alpha_h(u)\lambda_1' H'(u) + i\alpha_h'(u)(g_1' - c_1')/\alpha_h(u)\right]}{D}, \tag{44}$$

$$\frac{\partial u}{\partial \sigma_s} = -\frac{\omega(1-\omega)\gamma_s \hat{z}\alpha_h(u) \left[\lambda_1'\left(g_2' - c_2'\right) - \lambda_2'\left(g_1' - c_1'\right)\right]}{D}, \tag{45}$$

where we substitute $[\omega\lambda_1 + (1-\omega)\lambda_2] = i/\alpha_h(u)$, by (28). Therefore, when i is not too big, we have

$$\frac{\partial \hat{z}}{\partial \sigma_s} \simeq D, \frac{\partial q^1}{\partial \sigma_s} \simeq D, \frac{\partial q^2}{\partial \sigma_s} \simeq -D. \tag{46}$$

In general, the sign of $\partial u/\partial \sigma_s$ cannot be determined by the sign of D. The above analysis suggests that the effects of changing i or σ_s hinge on the sign of D, which is discussed as follows.

Analyzing the equilibrium conditions, we find that (q^1, q^2) are determined by $\hat{z}$ in (35) and (36), s_s is determined by u and q^2 in (29), and (28) and (33) are two equations about q^1, q^2 and u. Therefore, by (37), we can transform the equilibrium conditions into two equations, (28) and (33), and two variables $(u, \hat{z})$. Furthermore, we can define two curves from (28) and (33), RB (for "real balances") and LU (for "liquidity and unemployment"), in the $(u, \hat{z})$ space.

For the RB curve, we can get its slope, and its movement against changing σ_s by (28),

$$\frac{d\hat{z}}{du_{|RB}} = -\frac{\alpha_h'(u) g_1' g_2' \left[\omega\lambda_1 + (1-\omega)\lambda_2\right]}{\alpha_h(u)\left[\omega g_2' \lambda_1' + (1-\omega)\left(1+\gamma_s\sigma_s\right) g_1' \lambda_2'\right]} < 0, \tag{47}$$

$$\frac{d\hat{z}}{d\sigma_{s|RB}} = -\frac{(1-\omega)\gamma_s z g_1' \lambda_2'}{\omega g_2' \lambda_1' + (1-\omega)\left(1+\gamma_s\sigma_s\right) g_1' \lambda_2'} < 0, \tag{48}$$

where we use the results from (37). It implies that the RB curve is downward sloping

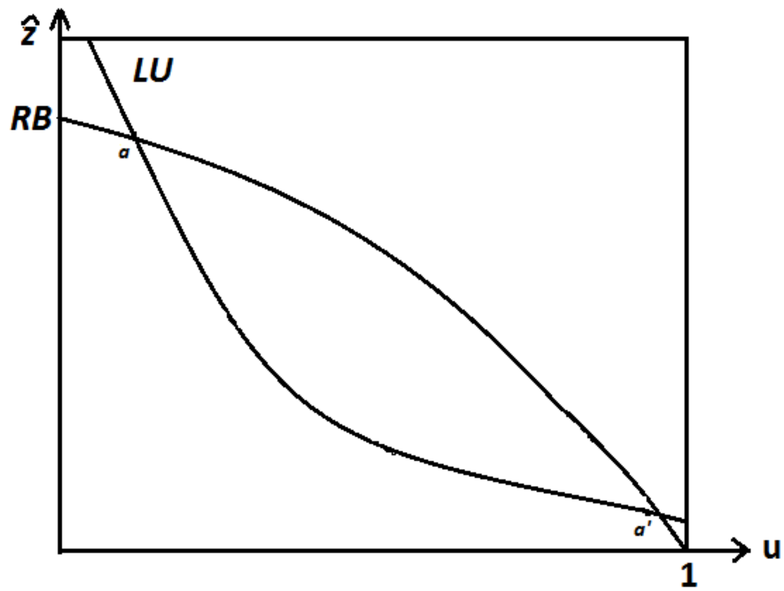

Figure 2: General Equilibrium by RB and LU

in $(u, \hat{z})$ space, and shifts down as σ_s increases. Similarly, for the LU curve, we have

$$\frac{du}{d\hat{z}}_{|LU} = \frac{\Omega}{g_1' g_2' H'(u)} < 0, \tag{49}$$

$$\frac{du}{d\sigma_s}_{|LU} = \frac{(1-\omega)\gamma_s z(g_2' - c_2')}{g_2' H'(u)} < 0. \tag{50}$$

The LU curve is also downward sloping in the $(u, \hat{z})$ space, and shifts down as σ_s increases.

In Figure 2, $\hat{z}$ goes to infinity when u is close to 0 on the LU curve. To see this, $H(u)$ approaches infinity when u is close to 0, by (33). It means that q^1, q^2 and $\hat{z}$ should become infinite. In addition, when $\hat{z}$ is close to 0, u is close to 0 on the LU curve. As for the RB curve, it has a positive intercept at $u = 0$, by (28), and $u = 1$ when $\hat{z} = 0$. As is shown in Figure 2, RB and LU could have multiple intersections, such as a and a'. However, if there exists a unique monetary equilibrium, it must be at point a.

An alternative way to see the sign of D is that it is *exactly* determined by the relative slopes of the LU and RB curves. For example, if RB cuts LU from below (LU is steeper than RB), we have $D < 0$. Otherwise, $D > 0$. Therefore, we can use the RB and LU curves to determine the sign of D, and then to study the effects of changing i or σ_s.

Changing i affects only the RB curve. That is, the effects of changing i can be shown by shifting the RB curve along the LU curve. As is shown in Figure 3, for the movement from point a to b, we have

$$\frac{\partial \hat{z}}{\partial i} < 0, \frac{\partial u}{\partial i} > 0. \tag{51}$$

This is consistent with (38) and (41) when $D < 0$. From (37), we can further obtain

$$\frac{\partial q^1}{\partial i} < 0, \frac{\partial q^2}{\partial i} < 0. \tag{52}$$

Again these results are consistent with (39)-(40), with $D < 0$. The results in (51) and (52) mean that higher inflation makes people economize on money, trade less in type-1 and type-2 meetings, and leads to higher unemployment. The effect on unemployment is also consistent with BMW, and the empirical evidence in Haug and King (2011).

For completeness of analysis, we also show the case $D > 0$ in Figure 3. Then changing i causes movement from a' to b', and we get the opposite results as in (51) and (52). At higher u, RB cuts LU from the above at point a', so that $D > 0$. Hence, higher i increases the real balances of money, and decrease u (the movement from point a' to b'). This is not a natural case. Therefore, we focus the case where there is a unique monetary equilibrium, i.e., $D < 0$, from now on.

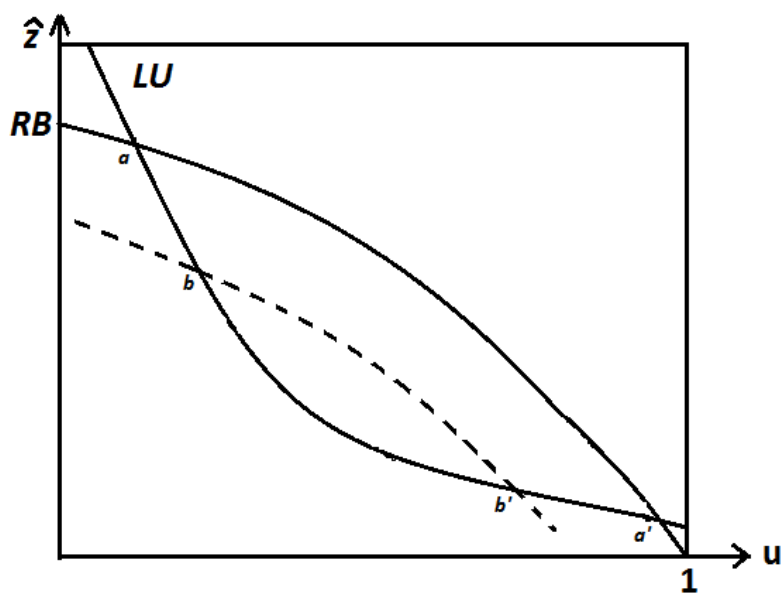

Figure 3: Effects of Changing i

For s_s and i_s, we have

$$\frac{\partial s_s}{\partial i} = -\frac{(1-\omega)\left\{\lambda_2 \alpha_h'(u)\Omega + (1+\gamma_s\sigma_s)\,\alpha_h(u) g_1' \lambda_2' H'(u)\right\}}{D},$$
$$\frac{\partial i_s}{\partial i} = -\frac{\omega\alpha_h'(u)\lambda_1\Omega + \omega\alpha_h(u)\lambda_1' g_2' H'(u) + \omega(1-\omega)\alpha_h^2(u)H'(u)\left[\lambda_1'(\upsilon_2' - g_2') - \lambda_2'(\upsilon_1' - g_1')\right]}{(1+s)^2 D}.$$

The effects of changing i on s_s and i_s are ambiguous, in general.

With $D < 0$, the effects of OMOs are ambiguous, in general, as in (42)-(45). It is not surprising if we check the movements of RB and LU by changing σ_s. We know both RB and LU shift down when increasing σ_s. Therefore, depending on the relative movements of RB and LU, the effects of changing σ_s are ambiguous on $\hat{z}$, u and other variables. However, when i is not too big, from (46), we have

$$\frac{\partial \hat{z}}{\partial \sigma_s} < 0,\ \frac{\partial q^1}{\partial \sigma_s} < 0,\ \frac{\partial q^2}{\partial \sigma_s} > 0.$$

Hence, OMOs have opposite effects on type-1 and type-2 meetings. It means an expansionary OMO (decreases σ_s), benefits households in type-1 meetings but hurt those in type-2 meetings. We label this as the *redistribution effect of OMOs*. Furthermore, from (28) and (29),

$$s_s = \frac{i\gamma_s(1-\omega)\lambda(q^2)}{\omega\lambda(q^1) + (1-\omega)\lambda(q^2)}.$$

Then we can derive

$$\frac{\partial s_s}{\partial \sigma_s} = \frac{i\gamma_s\omega(1-\omega)}{\left[\omega\lambda_1 + (1-\omega)\lambda_2\right]^2}\left[\lambda_1\lambda_2'\frac{\partial q^2}{\partial \sigma_s} - \lambda_2\lambda_1'\frac{\partial q^1}{\partial \sigma_s}\right] < 0,$$
$$\frac{\partial i_s}{\partial \sigma_s} = -\frac{1+i}{(1+s_s)^2}\frac{\partial s_s}{\partial \sigma_s} > 0,$$

when i is not too big.

As for the effects of OMOs on labor market, the *redistribution effect* explains why the effect of OMOs on labor market is ambiguous, even when i is not too big. From (33), $H(u)$ is the weighted surplus of sellers from type-1 and type-2 meetings, and $H'(u) < 0$. Therefore, with $\partial q^1/\partial\sigma_s < 0$ and $\partial q^2/\partial\sigma_s > 0$, the effect of an expansionary OMO on the weighted surplus of sellers from two types of meetings is ambiguous. Numerical results show the value of ω matters. When ω is big, the positive effect on type-1 meetings may dominate so that an expansionary OMO increases the total surplus of sellers, thus decrease u, i.e., $\partial u/\partial\sigma_s > 0$. When ω is small, the negative effect on type-2 meetings may dominate, so that an expansionary OMO increases u, i.e., $\partial u/\partial\sigma_s < 0$.

After the above analytical proof on the effects of changing σ_s, we can provide economic intuitions for the effects of OMOs. We focus on expansionary monetary policy when the central bank buys bonds to inject money, i.e., decreasing σ_s. When i is not too big, this type of OMOs increase $\hat{z}$ and q^1, but decrease q^2. This is because when the central bank purchases bonds, the higher demand for bonds will drive up the price of bonds. The higher price leads to a lower return on bonds, which makes bonds less attractive and hence q^2 lower. The lower return on bonds also encourages households to hold more money and therefore q^1 increases. Households consumer more in type-1 meetings but less in type-2 meetings. Overall, OMOs have an ambiguous impact on firms' expected gain S_f (or equivalently $H(u)$ in (33)), which further affects firms' entry decisions. Therefore, depending on the fractions of the two types of meetings, OMOs have an ambiguous effect on labor market. We summarize our analytical results as follows.

Proposition 2 *Consider the equilibrium with scarce bonds. When monetary equilibrium is unique, the effects of changing i are as follows,*

$$\frac{\partial\hat{z}}{\partial i} < 0,\ \frac{\partial q^1}{\partial i} < 0,\ \frac{\partial q^2}{\partial i} < 0,\ \frac{\partial u}{\partial i} > 0,\ \frac{\partial i_s}{\partial i} \gtrless 0,\ \frac{\partial s_s}{\partial i} \gtrless 0,$$

and the effects of changing σ_s are ambiguous in general, but when i is not too big,

$$\frac{\partial\hat{z}}{\partial\sigma_s} < 0,\ \frac{\partial q^1}{\partial\sigma_s} < 0,\ \frac{\partial q^2}{\partial\sigma_s} > 0,\ \frac{\partial i_s}{\partial\sigma_s} > 0,\ \frac{\partial s_s}{\partial\sigma_s} < 0,$$

while the sign of $\partial u/\partial\sigma_s$ is ambiguous.

5 Extension with Long-term Bonds

As discussed in the Introduction, after the GFC, central banks in the U.S. and other advanced economies have experienced the zero lower bound problem. It restricts the ability of central banks to conduct conventional monetary policy. In response, these central banks rely on unconventional monetary policy by large-scale purchases of long-term government bonds and private assets. The purpose is to *directly* affect long-term interest rates, to ease the conditions of financial markets, and thus to stimulate the economy.

In the baseline model, we allow the coexistence of money and short-term bonds, and analyze the effects of monetary policy by changing (i, σ_s). To model unconventional monetary policy since the GFC, we extend the baseline model to include long-term bonds. It means that households could hold a portfolio of money, short-term and long-term bonds. Firms in type-1 meetings still accept only money while firms in type-2 meetings accept all assets, including money and two types of bonds. We also model the pledgeability difference between two types of bonds, and examine the implications on monetary equilibrium. After solving for monetary equilibrium, we analyze the effects of OMOs on the economy's consumption and unemployment. In particular, unconventional monetary policy is modeled as OMOs where the central bank adjust the relative supply of long-term government bonds and money supply.

5.1 Extended Model

In Section 2, we define the pledgeability of the two types of bonds by γ_s, $\gamma_\ell \in (0, 1)$, nominal interest rates by (1) and (2), and two spreads by (3) and (4). Similar to (25), the nominal interest rate of long-term bonds should be no less than 0, but cannot exceed i:

$$0 \le i_\ell \le i. \tag{53}$$

With long-term bonds used in type-2 meetings, (13) is updated as

$$\begin{aligned} W_e^h(m, b_s, b_\ell) = \max_{\hat{m}, \hat{b}_s, \hat{b}_\ell} \{ & -\phi_m \hat{m} - \phi_s \hat{b}_s - \phi_\ell \hat{b}_\ell + \beta \left[\alpha_h S_h' + \phi_{m+}(\hat{m} + \hat{b}_s) + (\phi_{m+} + \phi_{\ell+})\hat{b}_\ell \right] \} \\ & + I_e + \phi_m(m + b_s) + (\phi_m + \phi_\ell) b_\ell, \end{aligned} \tag{54}$$

where $S_h' = \omega[\upsilon(q^1) - \phi_{m+}d^1] + (1-\omega)[\upsilon(q^2) - \phi_{m+}(d^2 + \mu_s) - (\phi_{m+}+\phi_{\ell+})\mu_\ell]$, with μ_ℓ denoting the amount of long-term bonds transferred in type-2 meetings. For firms, we update S_f as $S_f' = \omega[\phi_{m+}d^1 - c(q^1)] + (1-\omega)[\phi_{m+}(d^2 + \mu_s) + (\phi_{m+}+\phi_{\ell+})\mu_\ell - c(q^2)]$. Notice that S_f' includes the new payment from long-term bonds, $(\phi_{m+}+\phi_{\ell+})\mu_\ell$.

After adding long-term bonds, the government budget constraint is updated as

$$\phi_m (M - M_-) + \phi_s B_s + \phi_\ell B_\ell + T = \phi_m B_{s-} + (\phi_m + \phi_\ell) B_{\ell-} + u\kappa, \tag{55}$$

The difference between (55) and (22) is that the government's total expenditure and revenue include the value of long-term bonds in (55).Suppose that the central bank still commit to the policy described by (23) and (24). With long-term bonds, the central bank also commits to

$$\frac{(1+\phi_\ell/\phi_m)\,B_\ell}{M}=\sigma_\ell. \tag{56}$$

In steady states, we can rewrite (56) as the asset market clearing condition,

$$(1+\phi_\ell/\phi_m)\,\hat{b}_\ell=\sigma_\ell\hat{m}, \tag{57}$$

where $(1+\phi_\ell/\phi_m)\,\hat{b}_\ell$ is the total nominal value of long-term bonds handed over in the KW market. Hence, σ_ℓ is the ratio of the nominal value of long-term bonds to money supply, at the end of the current period. It can also be interpreted as the ratio of the long-term bonds (handed over in KW) to money supply. Suppose σ_ℓ is constant in steady states. Compared to the baseline model, the central bank can conduct *two types of OMOs*, i.e., changing the relative ratio of short-term bonds/long-term bonds to money supply. For example, when the central bank buys long-term bonds, it decreases the amount of long-term bonds held by the public, to inject money, i.e., decreasing σ_ℓ represents an expansionary monetary policy, while increasing σ_ℓ represents a contractionary policy. Therefore, the monetary policy parameters include (i,σ_s,σ_ℓ) in the extended model, with σ_s, $\sigma_\ell\in(0,+\infty)$.

5.2 Equilibrium

For equilibrium analysis, we follow similar steps as in the baseline model. We analyze equilibrium conditions in each market and then define general equilibrium. We begin with the bargaining solution in KW. For the goods market equilibrium, the bargaining solutions in two types of meetings are updated as follows.

Lemma 2 *In type-*1 *meetings, the bargaining solution is the same as (26), but in type-*2 *meetings,*

$$d^2=\hat{m},\; q^2=\begin{cases} g^{-1}[\phi_{m+}\hat{m}+\phi_{m+}\gamma_s\hat{b}_s+(\phi_{m+}+\phi_{\ell+})\gamma_\ell\hat{b}_\ell],\; \text{if } \mu_s=\gamma_s\hat{b}_s,\;\mu_\ell=\gamma_\ell\hat{b}_\ell \\ q^*\text{.....................................},\; \text{if } \mu_s<\gamma_s\hat{b}_s,\text{ or } \mu_\ell<\gamma_\ell\hat{b}_\ell,\text{ or both}\end{cases} \tag{58}$$

Proof 2 *As is in Proof 1, the cash constraint is always binding, i.e.,* $d^1=d^2=\hat{m}$*. In type-2 meetings, whether* $\mu_j\le\gamma_j b_j$ $(j=\{s,\ell\})$ *is binding depends on the returns of short-term and long-term bonds. When* $\mu_s=\gamma_s\hat{b}_s$*,* $\mu_\ell=\gamma_\ell\hat{b}_\ell$*,* q^2 *is solved from* $g(q^2)=\phi_{m+}(\hat{m}+\gamma_s\hat{b}_s)+(\phi_{m+}+\phi_{\ell+})\gamma_\ell\hat{b}_\ell$*. When neither of the two constraints is binding, or either of them is not binding,* $q^2=q^*$*.*

For now we focus on the case where $\mu_s = \gamma_s \hat{b}_s$, and $\mu_\ell = \gamma_\ell \hat{b}_\ell$. Then (35) still holds, but

$$g(q^2) = (1 + \gamma_s \sigma_s + \gamma_\ell \sigma_\ell)\hat{z}, \tag{59}$$

where we substitute $\hat{b}_\ell$ from (57). Substituting the bargaining solutions into (54), we can derive the interior solutions to the FOCs of (m, b_s, b_ℓ) as (28) and (29), plus

$$s_\ell = \alpha_h(u)(1-\omega)\gamma_\ell \lambda(q^2). \tag{60}$$

Similarly, the LHS of (60) s_ℓ is the marginal cost of spending 1 more unit of long-term bonds for households in type-2 meetings, while the RHS is the marginal benefit of spending 1 more unit of long-term bonds. Furthermore, from (29) and (60), we have

$$\frac{s_s}{s_\ell} = \frac{\gamma_s}{\gamma_\ell}. \tag{61}$$

Here we model long-term bonds as less pledgeable than short-term bonds (or with the same pledgeability), i.e., $\gamma_s \geq \gamma_\ell$. From (3)-(4) and (61), we can derive $i_s \leq i_\ell$. Hence, if short-term bonds are more pledgeable than long-term bonds, the nominal interest rate of long-term bonds should be higher than that of short-term bonds. This provides incentive for households to hold long-term bonds. Then the positive term premium is well justified, as in Williamson (2013) and Geromichalos et al.(2013).

The labor market equilibrium condition (32) still holds. We can substitute the bargaining solutions in (35) and (59) into (32) to derive (33). After solving the equilibrium conditions in different markets, we can define the stationary monetary equilibrium for the extended model.

Definition 2 *Given monetary policy parameters $(i, \sigma_s, \sigma_\ell)$, a stationary monetary equilibrium consists of $(\hat{m}, \hat{b}_s, \hat{b}_\ell, q^1, q^2, i_s, i_\ell, u)$ such that (i) given u, $(\hat{m}, \hat{b}_s, \hat{b}_\ell, q^1, q^2)$ solve (54) where (q^1, q^2) satisfies (26) and (58),and (i_s, i_ℓ) satisfies (1) and (2) ;(ii) given (q^1, q^2), u satisfies (32); and (iii) the asset market clears, and satisfies (34) and (57).*

The next step is to characterize different types of monetary equilibrium, which depends critically on the relative returns of the two types of bonds. Given $(i, \sigma_s, \sigma_\ell)$, similar to the baseline model, there are four types of equilibrium, although there are two subcases in each type of equilibrium, except in the Friedman Rule equilibrium.

5.2.1 Friedman Rule Equilibrium

Lastly, when $0 = i_s = i_\ell = i$, money and two types of bonds all have zero nominal returns. In this case, monetary policy is at the Friedman rule. The equilibrium allocation is $q^1 = q^2 = q^*$ and u is solved from (32). The Friedman rule equilibrium exists if and only if $\pi = \beta - 1$.

5.2.2 Liquidity Trap Equilibrium

When $0 = i_s \leq i_\ell < i$, we classify as the liquidity trap equilibrium. There are two subcases to consider.

Case 1: $0 = i_s < i_\ell < i$

In this subcase, households hold a portfolio of money and long-term bonds $(\hat{m}, \hat{b}_\ell)$, but no short-term bonds, i.e., $\hat{b}_s = 0$. This is because short-term bonds offer the same nominal return as money but are less acceptable than money. The necessary condition for this equilibrium to exist is $(i, \sigma_s, \sigma_\ell) \in$

$$\left\{(i, \sigma_s, \sigma_\ell) : 0 = i_s < i_\ell < i,\ \sigma_s = 0,\ 0 < \sigma_\ell < \sigma^*\right\}.$$

With $\hat{b}_s = 0$, (35),(28), (60) and (33) still hold, but not (29), and (59) becomes,

$$g(q^2) = (1 + \gamma_\ell \sigma_\ell)\hat{z}. \tag{62}$$

From these equilibrium conditions, we can get the equilibrium $(\hat{m}, \hat{b}_\ell, q^1, q^2, i_\ell, u)$. With $i_s = 0$, the economy is in the liquidity trap. But the central bank can still conduct money policy by setting (i, σ_ℓ). Except changing i, the central bank can conduct OMOs by changing σ_ℓ. In fact, this case is quite similar to the equilibrium with scarce bonds in the baseline model, except that long-term bonds replace short-term bonds, and thus σ_ℓ replaces σ_s. Therefore, for simplicity, we omit the proof, and just provide the analytical results as follows.

Proposition 3 *Consider Case 1 of the liquidity trap equilibrium. When monetary equilibrium is unique, the effects of changing i are as follows,*

$$\frac{\partial \hat{z}}{\partial i} < 0,\ \frac{\partial q^1}{\partial i} < 0,\ \frac{\partial q^2}{\partial i} < 0,\ \frac{\partial u}{\partial i} > 0,\ \frac{\partial i_\ell}{\partial i} \gtrless 0,\ \frac{\partial s_\ell}{\partial i} \gtrless 0,$$

and the effects of changing σ_ℓ are ambiguous in general, but when i is not too big,

$$\frac{\partial \hat{z}}{\partial \sigma_\ell} < 0,\ \frac{\partial q^1}{\partial \sigma_\ell} < 0,\ \frac{\partial q^2}{\partial \sigma_\ell} > 0,\ \frac{\partial i_\ell}{\partial \sigma_\ell} > 0,\ \frac{\partial s_\ell}{\partial \sigma_\ell} < 0,$$

while the sign of $\partial u/\partial \sigma_\ell$ is ambiguous.

Case 2: $0 = i_s = i_\ell < i$

In this subcase, households hold only money. The economy becomes a pure monetary economy, as in BMW. Therefore, this case is the same as the liquidity trap equilibrium of the baseline model. This equilibrium exists if and only if $(i, \sigma_s, \sigma_\ell) \in$

$$\{(i, \sigma_s, \sigma_\ell) : i > 0, \sigma_s = \sigma_\ell = 0\}.$$

5.2.3 Equilibrium with Plentiful Bonds

When either i_s or i_ℓ is equal to i, or both equal to i, it is costless to hold some bonds. In this case, the constraints of type-2 meetings may not be binding, in (58). Therefore, households may choose to hold an infinite amount of the bonds that have the high return. We label all of these cases as the equilibrium with plentiful bonds. In addition, we have $q^2 = q^*$. OMOs become irrelevant. We provide details as follows.

Case 1: $0 = i_s < i_\ell = i$

In this subcase, households will hold the portfolio $(\hat{m}, \hat{b}_\ell)$, with money and an infinite amount of long-term bonds, but no short-term bonds. With $i_s = 0$, short-term bonds have the same nominal return as money. Since they are less acceptable than money (only accepted in type-2 meetings), and less pledgeable as money ($\gamma_s < 1$), households will not hold short-term bonds. It is costless to hold long-term bonds since $i_\ell = i$. Households may hold an infinite amount of long-term bonds and consume q^* in type-2 meetings. This equilibrium exists if and only if $(i, \sigma_s, \sigma_\ell) \in$

$$\left\{(i, \sigma_s, \sigma_\ell) : 0 = i_s < i_\ell = i,\ \sigma_s = 0,\ \sigma_\ell > \sigma^*\right\}.$$

Case 2: $0 < i_s = i_\ell = i$

With $i_s = i_\ell$, we have $\gamma_s = \gamma_\ell$. In this case, short-term and long-term bonds become perfect substitutes, with the same acceptability and pledgeability. Households hold a portfolio of money and bonds, but are indifferent between short-term bonds and long-term bonds. This equilibrium exists if and only if $(i, \sigma_s, \sigma_\ell) \in$

$$\left\{(i, \sigma_s, \sigma_\ell) : 0 < i_s = i_\ell = i,\ \sigma_s > 0,\ \sigma_\ell > 0, \text{ and } \gamma_s\sigma_s + \gamma_\ell\sigma_\ell > \sigma^*\right\}.$$

5.2.4 Equilibrium with Scarce Bonds

When $0 < i_s < i_\ell < i$, households hold a portfolio of money, short-term and long-term bonds. This is also the case where all constraints in type-2 meetings are binding.

Besides Case 1 in the liquidity trap equilibrium, this is another case where OMOs matter. It is a complicated but interesting case in the extended model. To get equilibrium $(\hat{z}, q^1, q^2, i_s, i_\ell, u)$, we have the equilibrium conditions in (35), (59), (28), (29), (60) and (33).[5] A necessary condition for this equilibrium to exist is that $(i, \sigma_s, \sigma_\ell) \in$

$$\left\{(i, \sigma_s, \sigma_\ell) : 0 < i_s < i_\ell < i,\ \sigma_s > 0,\ \sigma_\ell > 0, \text{ and } \gamma_s\sigma_s + \gamma_\ell\sigma_\ell < \sigma^*\right\}.$$

Taking full derivation against (35), (59), (28), and (33), we can analyze the effects of inflation and OMOs. For the effects of inflation (changing i), we have

$$\frac{\partial z}{\partial i} = -\frac{g_1' g_2' H'(u)}{D_e} \simeq D_e, \tag{63}$$
$$\frac{\partial q^1}{\partial i} = -\frac{g_2' H'(u)}{D_e} \simeq D_e, \tag{64}$$
$$\frac{\partial q^2}{\partial i} = -\frac{(1 + \gamma_s\sigma_s + \gamma_\ell\sigma_\ell)\, g_1' H'(u)}{D_e} \simeq D_e, \tag{65}$$
$$\frac{\partial u}{\partial i} = -\frac{\omega g_2' (g_1' - c_1') + (1-\omega)(1 + \gamma_s\sigma_s + \gamma_\ell\sigma_\ell)\, g_1' (g_2' - c_2')}{D_e} \simeq -D_e, \tag{66}$$

where $D_e = -\alpha_h'(u)\left[\omega\lambda_1 + (1-\omega)\lambda_2\right]\Omega_e - \alpha_h(u)H'(u)\left[\omega g_2'\lambda_1' + (1-\omega)(1 + \gamma_s\sigma_s + \gamma_\ell\sigma_\ell)\, g_1'\lambda_2'\right]$, and $\Omega_e = \left[\omega g_2'(g_1' - c_1') + (1-\omega)(1 + \gamma_s\sigma_s + \gamma_\ell\sigma_\ell)\, g_1'(g_2' - c_2')\right] > 0$. The results here are quite similar to the equilibrium of scarce bonds in the baseline model. It is possible that $D_e > 0$ or $D_e < 0$. The effects of changing i are all determined by the sign of D_e.

For the effects of OMOs, there are two parameters, σ_s and σ_ℓ so that we need to consider the effects by changing them in turn. When the central bank buys or sells short-term government bonds to conduct OMOs, it changes σ_s, and the effects are

$$\frac{\partial z}{\partial \sigma_s} = \frac{(1-\omega)\gamma_s z g_1' \left[\alpha_h(u)\lambda_2' H'(u) + i\alpha_h'(u)(g_2' - c_2')/\alpha_h(u)\right]}{D_e}, \tag{67}$$
$$\frac{\partial q^1}{\partial \sigma_s} = \frac{(1-\omega)\gamma_s z \left[\alpha_h(u)\lambda_2' H'(u) + i\alpha_h'(u)(g_2' - c_2')/\alpha_h(u)\right]}{D_e}, \tag{68}$$
$$\frac{\partial q^2}{\partial \sigma_s} = -\frac{\omega\gamma_s z \left[\alpha_h(u)\lambda_1' H'(u) + i\alpha_h'(u)(g_1' - c_1')/\alpha_h(u)\right]}{D_e}, \tag{69}$$
$$\frac{\partial u}{\partial \sigma_s} = -\frac{\omega(1-\omega)\gamma_s z \alpha_h(u)\left[\lambda_1'(g_2' - c_2') - \lambda_2'(g_1' - c_1')\right]}{D_e}, \tag{70}$$

[5] We do not include the equilibrium b_s, b_ℓ, since they are both related with m (equivalently, $\hat{z}$) by (34) and (57).

where we substitute $[\omega\lambda_1 + (1-\omega)\lambda_2] = i/\alpha_h(u)$, by (28). Therefore, when i is not too big, we have

$$\frac{\partial z}{\partial \sigma_s} \simeq D_e,\ \frac{\partial q^1}{\partial \sigma_s} \simeq D_e,\ \frac{\partial q^2}{\partial \sigma_s} \simeq -D_e. \tag{71}$$

Notice that $\partial u/\partial \sigma_s$ cannot be determined by the sign of D_e. Again these results are quite similar to the baseline model.

A change in σ_ℓ means that the central bank conducts OMOs by buying or selling long-term bonds. The effects of changing σ_ℓ are

$$\frac{\partial z}{\partial \sigma_\ell} = \frac{(1-\omega)\gamma_\ell z g_1' \left[\alpha_h(u)\lambda_2' H'(u) + i\alpha_h'(u)(g_2'-c_2')/\alpha_h(u)\right]}{D_e}, \tag{72}$$

$$\frac{\partial q^1}{\partial \sigma_\ell} = \frac{(1-\omega)\gamma_\ell z \left[\alpha_h(u)\lambda_2' H'(u) + i\alpha_h'(u)(g_2'-c_2')/\alpha_h(u)\right]}{D_e}, \tag{73}$$

$$\frac{\partial q^2}{\partial \sigma_\ell} = -\frac{\omega\gamma_\ell z \left[\alpha_h(u)\lambda_1' H'(u) + i\alpha_h'(u)(g_1'-c_1')/\alpha_h(u)\right]}{D_e}, \tag{74}$$

$$\frac{\partial u}{\partial \sigma_\ell} = -\frac{\omega(1-\omega)\gamma_\ell z\alpha_h(u) \left[\lambda_1'(g_2'-c_2') - \lambda_2'(g_1'-c_1')\right]}{D_e}, \tag{75}$$

where we substitute $[\omega\lambda_1 + (1-\omega)\lambda_2] = i/\alpha_h(u)$, by (28). Therefore, when i is not too big, we have

$$\frac{\partial z}{\partial \sigma_\ell} \simeq D_e,\ \frac{\partial q^1}{\partial \sigma_\ell} \simeq D_e,\ \frac{\partial q^2}{\partial \sigma_\ell} \simeq -D_e. \tag{76}$$

But $\partial u/\partial \sigma_\ell$ cannot be determined by the sign of D_e, even when i is not too big.

Therefore, similar to the baseline model, the effects of changing i , σ_s or σ_ℓ hinge on the sign of D_e, which can still be discussed by the movements of RB and LU curves as before. Since it is quite similar to the baseline model, we abstract from the detailed analysis, and summarize the analytical results as follows.

Proposition 4 *Consider the equilibrium with scarce bonds. When monetary equilibrium is unique, the effects of changing i are as follows,*

$$\frac{\partial z}{\partial i} < 0,\ \frac{\partial q^1}{\partial i} < 0,\ \frac{\partial q^2}{\partial i} < 0,\ \frac{\partial u}{\partial i} > 0, \frac{\partial i_s}{\partial i} \gtrless 0,\ \frac{\partial s_s}{\partial i} \gtrless 0,\ \frac{\partial i_\ell}{\partial i} \gtrless 0,\ \frac{\partial s_\ell}{\partial i} \gtrless 0,$$

and the effects of changing σ_s or σ_s are ambiguous, in general, but when i is not too big,

$$\frac{\partial z}{\partial \sigma_s} < 0,\ \frac{\partial q^1}{\partial \sigma_s} < 0,\ \frac{\partial q^2}{\partial \sigma_s} > 0,\ \frac{\partial i_s}{\partial \sigma_s} > 0,\ \frac{\partial s_s}{\partial \sigma_s} < 0,\ \frac{\partial i_\ell}{\partial \sigma_s} > 0,\ \frac{\partial s_\ell}{\partial \sigma_s} < 0,$$
$$\frac{\partial z}{\partial \sigma_\ell} < 0,\ \frac{\partial q^1}{\partial \sigma_\ell} < 0,\ \frac{\partial q^2}{\partial \sigma_\ell} > 0,\ \frac{\partial i_\ell}{\partial \sigma_\ell} > 0,\ \frac{\partial s_\ell}{\partial \sigma_\ell} < 0,\ \frac{\partial i_s}{\partial \sigma_\ell} > 0,\ \frac{\partial s_s}{\partial \sigma_\ell} < 0,$$

while the signs of $\partial u / \partial \sigma_s$ and $\partial u / \partial \sigma_\ell$ are ambiguous.

5.3 Discussion

In the extension, we introduce long-term bonds to the model, in addition to money and short-term bonds. The central bank now can change either σ_s or σ_ℓ to conduct OMOs. Among different types of monetary equilibrium, we are particularly interested in Case 1 of the liquidity trap equilibrium, and the equilibrium with scarce bonds. In Case 1 of the liquidity trap equilibrium, the nominal interest rate of short-term bonds is zero. It means there is little room for the central bank to lower the short-term interest rate through OMOs.[6] It resembles the situation of hitting the zero lower bound in the U.S., Japan and some European countries. In this case, the central bank can adjust the supply of long-term bonds to stimulate the economy. In particular, central bank's purchase of long-term bonds bids up the price of long-term bonds and hence lowers its return. OMOs with long-term bonds still have a redistribution effect, since it benefits the consumption in type-1 meetings but hurts that in type-2 meetings. The overall effect on labor market is ambiguous because firms' profits from goods trading may or may not increase. This transmission channel of OMOs is similar to our finds from the baseline model. In some sense, we argue that unconventional monetary policy is "*conventional*" monetary policy during "*unconventional*" time.

In the equilibrium of scarce bonds, we find that both conventional and unconventional monetary policies are effective. Households hold portfolios of money, short-term bonds and long-term bonds. The central bank can adjust either σ_s or σ_ℓ to affect the economy. It resembles economies during normal times, e.g., the U.S. prior to the GFC. Historically, the Fed did long-term bonds transactions between 1942 and 1951, as documented by D'Amico et al. (2012).[7] It demonstrates how the central bank could directly affect the long-term interest rate by transacting in the long-term securities' markets. The equilibrium with scarce bonds allows us to examine effects of both conventional and unconventional monetary policies. In addition to the transmission channel we have emphasized, we find that changing the supply of one type of bonds has a general equilibrium effect on the return on the other type of bonds.

[6] As is mentioned before, ECB and the Swiss National Bank did cut the short-term target rates to negative in 2014. But ECB has decided to resort to unconventional monetary policy, starting in March, 2015, after it realizes imposing the negative interest rate may cause deflation, and does not really stimuate the real economy.

[7] D'Amico et al. (2012) provide a historial review of the Fed's operations in long-term government bonds in the post-war period before 2008. It explains how long-term bonds entered the portfolio of the Fed, and how their ratio in the total assets of the Fed changed historically.

On the modeling of OMOs with long-term bonds, we use the ratio of long-term bonds (handed over in KW) to money supply. An alternative way is to follow the baseline model and use the ratio of long-term bonds to money supply. As is shown in RWX, this approach of modeling OMOs with long-term bonds yields the same effect of OMOs as that with short-term bonds. In our model, the existence of labor market makes this approach less tractable. Therefore, we slightly modify the definition of OMOs with long-term bonds, although we plan to explore more implications from the approach in RWX.

6 Conclusion

We build models where money and bonds coexist to examine the effects of monetary policy on macroeconomic performance such as consumption and unemployment. Our model includes explicit modeling of labor market, goods markets, and asset liquidity. In the baseline model, money and short-term government bonds coexist to facilitate goods trading. Money and short-term bonds differ in their acceptability and pledgeability: money is accepted everywhere and more pledgeable than short-term bonds. We show that monetary policy, including adjusting inflation and OMOs, has effects on macroeconomic activities.

We then extend the model to add long-term bonds. Long-term bonds differ from short-term bonds in that the former are less pledgeable in goods market and hence offer higher returns than the latter. The central bank can conduct either conventional monetary policy by adjusting its holdings of short-term bonds, or unconventional monetary policy by adjusting its holdings of long-term bonds. We use the extended model to analyze how unconventional monetary policy affects consumption and unemployment, particularly when the economy is in the liquidity trap. Our theoretical results show OMOs have a redistribution effect on the two types of meetings in the goods market. The effect of OMOs on unemployment is ambiguous, which depends on the fractions of the two types of meetings.

In terms of future work, there are a few directions we can pursue. Our model predicts the redistribution effect associated with OMOs. It would be useful if we can test for this prediction empirically. Theoretically, we can consider adding private asset purchase by the central bank to the model, to show the comprehensive effects of unconventional monetary policy. We may also incorporate fiscal policy in our model to explore the interaction between government debt and unconventional monetary policy. All of this is left for future work.

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