Frictional Capital Reallocation with Ex Post Heterogeneity*

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Abstract

We study frictional markets for capital reallocation based on ex post heterogeneity, where firms with similar portfolios realize different shocks. For various specifications, including random search and bargaining plus directed search and posting, results are provided on existence, uniqueness, efficiency, and the effects of monetary/fiscal policy. We show, e.g., higher nominal interest rates can lower or raise investment, and can be desirable even while hindering reallocation. The model captures several stylized facts, e.g., misallocation is countercyclical, while capital's price and reallocation are procyclical. We also ask if productivity dispersion is a good measure of inefficiency or frictions. While the theory speaks to many issues, it tractably reduces to two equations, for accumulation and reallocation, resembling the curves commonly taught in undergraduate macro.

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1 Introduction

This paper studies dynamic general equilibrium models where capital is accumulated in primary markets and reallocated in frictional secondary markets. What drives reallocation is that firms are ex post heterogeneous due to productivity shocks. Thus, efficient economic performance requires getting the right amount of investment over time, plus getting existing capital into the hands of those best able to use it at a point in time. While macroeconomics traditionally concentrated on the former, the latter is currently receiving much attention, and of course the two are intimately related: the ease with which used capital can be retraded affects incentives for the accumulation of new capital, just like the attributes of secondary markets for houses, cars and other assets influence supply and demand in primary markets.

One motivation is that reallocation is big, with purchases of used capital constituting 25% to 30% of total investment (Eisfeldt and Rampini 2006; Cao and Shi 2016; Cui 2017; Dong et al. 2016). In fact, given their data these studies do not include mergers, they ignore smaller firms and those that are not publicly traded, and they include purchases but exclude rentals.[1] Given capital reallocation is sizable, we are interested in how it depends on fiscal and monetary policy. On the former, taxation is shown to be important for capital accumulation, if not reallocation, by Cooley and Hansen (1992), Chari et al. (1994), McGrattan et al. (1997), McGrattan (2012), etc. On the latter, precedents for studying inflation and accumulation, if not reallocation, include Tobin (1965), Sydrauski (1967), Stockman (1981), Cooley and Hansen (1989), etc., although they all use reduced-form (e.g., CIA or MUF) models, while we take a more microfounded approach.[2]

[1] These facts, like our model, concern reallocation across firms; in principle one can also consider movement of capital within firms (Giroud and Mueller 2015), across sectors (Ramey and Shapiro 1998) or between countries (Caselli and Feyrer 2007).

[2] Our methods follow the New Monetarist literature surveyed by Lagos et al. (2017) and Rocheteau and Nosal (2017), but we study firms trading inputs while those papers study households trading goods, with a few exceptions. Work on capital based on Lagos and Wright (2005),

As for *frictional* reallocation, many people argue that real-world capital markets as neither perfectly competitive nor frictionless, e.g., Kurman and Petrosky-Nadeau (2007), Gavazza (2010,2011*a,b*), Kurman (2014), Ottonello (2015), Cao and Shi (2016), Kurman and Rabinovitz (2018) and Horner (2018). Imperfections include adverse selection, financial constraints, the difficulty of finding an appropriate counterparty, and holdup problems due to bargaining. We downplay adverse selection (see Li and Whited 2014, Eisfeldt and Rampini 2008 and references therein) to concentrate on liquidity, search and bargaining. Thus, capital is traded in an OTC (over-the-counter) market featuring bilateral exchange and bargaining, as in equilibrium search theory, and the use of assets in facilitating trade, as in modern monetary economics.

Further on motivation, Ottonello (2015) compares capital markets with and without search. He argues the former fit the facts better and generate more interesting propagation: in his model, financial shocks account for 33% of fluctuations with search, and 1% without. Horner (2018) shows that vacancy rates for industrial, retail and office real estate resemble unemployment rates, suggesting search may be as important for real estate capital as it is for labor. He also shows rents on similar structures vary considerably, inconsistent with Walrasian theory. Several studies on the market for aircraft (Pulvino 1998; Gilligan 2004; Gavazza 2011*a,b*) find used sales are thrice new sales and prices vary inversely with search time. Gavazza (2011a) in particular shows how market thickness affects trading frequency, average utilization, utilization dispersion, average price, and price dispersion. He also emphasizes the importance of specificity: capital

like our model, includes Aruoba and Wright (2003), Lagos and Rocheteau (2008), Aruoba et al. (2011) and Andolfatto et al. (2016). Other work includes Shi (1998,1999*a,b*), Shi and Wang (2006), Menner (2006) and Berentsen et al. (2011), based on Shi (1997), and Molico and Zhang (2006), based on Molico (2006). Also related are models of firms trading ideas, e.g., Silveira and Wright (2010), Chiu and Meh (2011) and Chiu et al. (2017). Lagos and Zhang (2018) is similar, too, except there agents trade assets in fixed supply (as opposed to reproducable capital) and get idiosyncratic preference shocks over dividends (as opposed to technology shocks). Still, that model also emphasizes feedback between primary and secondary asset markets.

is often customized to a firm, making it hard to find another with similar needs. This all suggests pursuing a search-based approach.

Now any model of reallocation builds on gains from trade, with capital flowing from lower- to higher-productivity firms, as it does in the data (Maksimovic and Phillips 2001; Andrade et al. 2001; Schoar 2002). In previous work (Wright et al. 2017) we studied ex ante heterogeneity, where only some firms can get capital in the primary market, leading to obvious gains from trade in the secondary market. Here instead all firms can build capital in the primary market, but then receive idiosyncratic productivity shocks. This ex post heterogeneity is more natural and more in line with other research, It also lets us dispense with a few awkward assumptions in the earlier model.[3] Moreover, the new model is very tractable, reducing to two equations for investment and reallocation, that let us easily demonstrate the effects of policy.

As a preview, conditional on investment, reallocation is efficient at the Friedman rule $\iota = 0$, where ι is the nominal interest rate. Yet investment can be too high or low, depending on various factors, including the capital income tax τ. At $\tau = \iota = 0$, with random search and bargaining, there is a Hosios (1990) condition for bargaining power θ, such that equilibrium is efficient iff $\theta = \theta^*$ (which is not true for ex ante heterogeneity, as explained below). Alternatively, with directed search and price posting, equilibrium is efficient at $\tau = \iota = 0$, with no restriction on θ, since there is no bargaining. Now consider a second-best world where, e.g., policy may have a role due to bargaining with $\theta \neq \theta^*$. We prove this strong result: given $\tau = 0$, the optimal monetary policy is $\iota^* > 0$ if θ is *either* too low or too high. This is interesting because the Friedman rule is optimal in very many

[3] When a firm must get all its capital in the frictional market, it wants to acquire a lot when it can, lest it cannot get any later; the old model awkwardly precluded this by assuming these firms can only store capital for 1 period. Relatedly, it assumed they must find new trading partners each period, ruling out long-term relationships. This is less of an issue here, as firms have less need for long-term relationships when they can get always capital in primary markets; there is still some benefit to enduring relationships, given search frictions, but we can let the probability of finding a counterparty be sufficiently high that this is unimportant.

models. Moreover, given $\iota = 0$, the optimal policy is $\tau^* > 0$ if θ is too low and $\tau^* < 0$ if θ is too high; hence, monetary and fiscal policy are not symmetric.

The theory also provides insights into the measurement of misallocation. It might seem that a natural indicator of capital mismatch is dispersion in productivity across firms, but we show that various measures of productivity dispersion are generally imperfect indicators of the underlying frictions or welfare/output gaps. Higher ι, e.g., captures greater financial frictions, but can reduce dispersion measures (similar results hold if we replace higher ι by lower debt limits, lower arrival rates or higher taxes). It also provides insights into observations deemed important in the literature. One such observation is that reallocation is procyclical but capital mismatch appears countercyclical (Eisfeldt and Rampini 2006; Cao and Shi 2016). Here, in good times, there may well be less incentive to reallocate capital due to lower productivity dispersion, but there is also more capital, so reallocation can be greater. Similarly, the price of reallocated capital can be higher in goods times (Lanteri 2016), as can the ratio of spending on used capital to total investment (Cui 2016).

Further on the literature, Ottonello (2015), Cao and Shi (2016), Dong et al. (2016), Kurman and Rabinovitz (2018) and Horner (2018) are recent papers on capital and search, but with many differences, e.g., they are nonmonetary models. Rocheteau et al. (2017) study a monetary model, but again with key differences, e.g., their firms buy capital from competitive suppliers, they do not trade with each other in frictional markets. Also relevant is empirical work on productivity like Hsieh and Klenow (2009), Buera et al. (2011), Midrigan and Xu (2014), Cooper and Schott (2016), Ai et al. (2015) and David and Venkateswaran (2017). Our model is broadly consistent with their findings. Buera et al. (2011), e.g., find financial frictions explain empirical regularities by distorting the capital allocation across firms, even if self-financing reduces this problem. Our emphasis on liquidity is meant to capture this.

The paper is also related to research on OTC financial markets. Duffie et al. (2005), e.g., is a search model where agents trade assets due to idiosyncratic preference shocks. There are several differences: The asset here is neoclassical capital, instead of "trees" bearing "fruit." We have capital (and labor) used to produce numeraire goods, as in standard growth theory, instead of assuming agents consuming "fruit" directly. Our agents face a genuine liquidity problem because they must pay at least in part with retained earnings held at low yield, instead of transferable utility. Moreover, Duffie et al. (2005) restrict asset positions to $\{0, 1\}$, while we do not, making our setup perhaps more comparable to Lagos and Rocheteau (2009). Having said all that, the ideas here are similar to those in the literature on frictional markets for financial assets.

To connect with other macro models, shutting down firms' idiosyncratic shocks here eliminates the need for secondary markets and liquidity, reducing to a standard Real Business Cycle model (literally, the one in Hansen 1985). Thus, instead of saying we extend monetary theory to study capital reallocation, one can say we extend more mainstream macro to incorporate idiosyncratic shocks, frictional secondary markets and liquidity considerations. By comparison, Asker et al. (2014) present a macro model where it takes one period to build capital, with productivity dispersion arising from this lag. What is missing relative to our setup is a market for retraading capital after shocks are realized, where search, bargaining and liquidity all play roles.

In what follows, first, Section 2 describes the environment. Then Sections 3 and 4 discuss equilibrium with credit and with money, respectively. Section 5 analyzes a particularly convenient parameterization. Section 6 takes up directed search and price posting, as opposed to random search and bargaining. Section 7 presents numerical results to show the theory can match some facts and to discuss how good or bad alternative measures of productivity dispersion are at capturing misallocation, frictions and welfare. Section 8 concludes.

2 Environment

Time is discrete and continues forever. As shown in Figure 1, at each date t two markets convene sequentially: a frictional decentralized market, or DM; and a frictionless centralized market, or CM. This alternating market structure, adapted from Lagos and Wright (2005), is ideal for our purposes because the CM and DM correspond nicely to primary and secondary capital markets. In the CM, agents consume a numeraire good x, supply labor hours h and accumulate capital k as in standard growth theory. Then in the DM, rather than households trading consumption goods, as in most of the related literature, we have firms trading capital (although, for convenience, sometimes we call them households that own firms rather than firms per se). All agents (firm owners) have utility $U(x,h) = u(x) - Ah$, where u is strictly increasing, concave and satisfies Inada conditions. They all discount between the CM and the next DM at rate $\beta \in (0,1)$, but not between the DM and CM, without loss of generality.

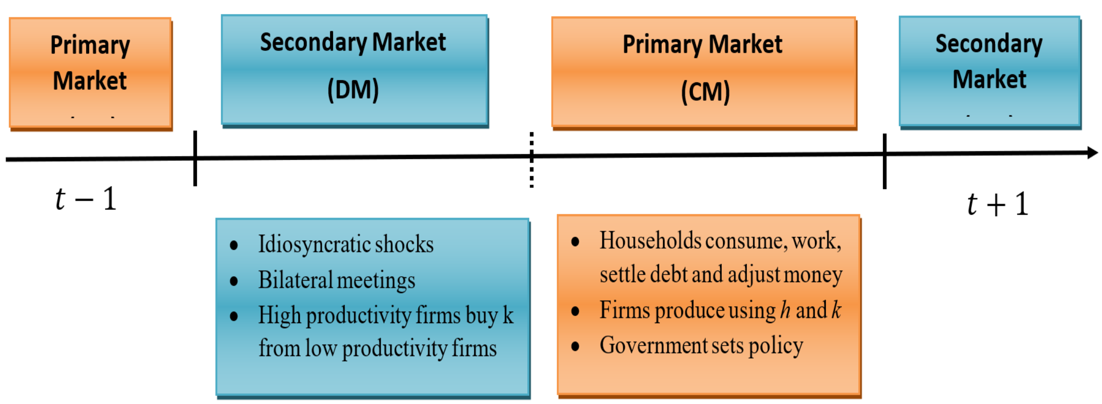


Figure 1: Time Line

For initial conditions, start in the DM with all agents holding k_0. Then each firm realizes an idiosyncratic productivity shock $\varepsilon \in [0, \bar{\varepsilon}]$, with CDF $G(\varepsilon)$. While persistence is interesting, for now the shocks are i.i.d. to focus on ex post heterogeneity.[4] Thus, firms have a technology $F(k,h,\varepsilon)$ in the subsequent CM, where higher ε entails more output for any (k,h), and $F(\cdot)$ displays decreasing returns to scale (see fn. 6 below). Also, F is strictly increasing, strictly concave, and has continuous second-order derivatives. While we do not generally require these conditions, sometimes it is convenient to also impose $F_1(k,h,\varepsilon) \to 0$ as $k \to \infty$ or $F_2(k,h,\varepsilon) \to 0$ as $h \to \infty$. In any case, the ε shocks generate gains from retrading k in the DM, which here features random pair-wise meetings, with α denoting the probability any agent meets a counterparty.[5] Each meeting is characterized by a state $\mathbf{s} = (k_b, \varepsilon_b, k_s, \varepsilon_s)$, where k_b and ε_b are the capital and productivity of the buyer, while k_s and ε_s are the same for the seller. Given i.i.d. shocks, on the equilibrium path all agents start the DM with the same capital, so $k_b = k_s$ and hence the buyer is the one with the better shock, $\varepsilon_b > \varepsilon_s$.

The first task is to discuss efficiency. For any firm in the CM with (k,ε), as long as $h > 0$, it is given by $h^*(k,\varepsilon)$ where

$$F_2[k, h^*(k,\varepsilon), \varepsilon]u'(x) = A. \tag{1}$$

Total hours aggregates this across firms, and if there is a time constraint for a household, say $h \in [0,1]$, for most of what we do it is assumed slack. It is also obvious that when two firms meet in the DM the one with the lower ε should give q units of k to the other one, where q equates marginal products, subject to $q \leq k$. This constraint can bind.[6] However, if we impose $F_1(k,h,\varepsilon) \to \infty$ as

[4] It is easy to include a permanent shock where, e.g., with probability γ_0 each period a firm transits to an absorbing state where output is 0. This is a very easy way to get exit, especially if firms that shut down are simply replaced by new new ones. It is only slightly harder to let $\varepsilon = \varepsilon_T + \varepsilon_P$ where ε_T is the transitory shock in our benchmark more and ε_P is persistant but not permanent and not necessarily such that the firm exists.

[5] As usual in search theory, we do not take the notion of meetings literally as firms randomly "bumping into each other" or "being in contact with each other;" it is rather an attempt to tractably capture the difficulty of realizing all gains from trade, something missing in classical equilibrium theory.

[6] As Jovanovic and Rousseau (2002) say: "Used equipment and structures sometimes trade unbundled in that firm 1 buys a machine or building from firm 2, but firm 2 continues to exist. At other times, firm 1 buys firm 2 and thereby gets to own all of firm 2's capital. In both

$k \to 0$, it is slack, and $q = q^*(\mathbf{s})$ satisfies[7]

$$F_1[k+q, h^*(k+q, \varepsilon_b), \varepsilon_b] = F_1[k-q, h^*(k-q, \varepsilon_s), \varepsilon_s]. \quad (2)$$

With these results in hand, consider a planner choosing a path for investment to maximize expected utility for the representative agent, subject to the search frictions, an initial k_0, and resource feasibility given that government takes g_t units of numeraire each period. The planner problem can be written as

$$\begin{aligned} W^*(k_0) &= \max_{k_{t+1}} \sum_{t=0}^{\infty} \beta^t [u(x_t) - Ah_t] \qquad (3) \\ \text{st } x_t &= y_t + (1-\delta) k_t - g_t - k_{t+1} \\ y_t &= (1-\alpha) \int_0^{\infty} F[k_t, h^*(k_t, \hat{\varepsilon}), \hat{\varepsilon}] dG(\hat{\varepsilon}) \\ &\quad + \alpha \int_{\{\hat{\varepsilon} \geq \tilde{\varepsilon}\}} F\{k_t + q^*(\hat{\mathbf{s}}), h^*[k_t + q^*(\hat{\mathbf{s}})], \hat{\varepsilon}\} dG(\tilde{\varepsilon}) dG(\hat{\varepsilon}) \\ &\quad + \alpha \int_{\{\hat{\varepsilon} < \tilde{\varepsilon}\}} F\{k_t - q^*(\tilde{\mathbf{s}}), h^*[k_t - q^*(\tilde{\mathbf{s}})], \hat{\varepsilon}\} dG(\tilde{\varepsilon}) dG(\hat{\varepsilon}), \end{aligned}$$

where aggregate output y_t includes production by the $1-\alpha$ measure of firms that did not have a DM meeting, the α measure that had a meeting and increased their k, and the α measure that had a meeting and decreased their k

markets, the traded capital gets a new owner." The latter can be called an aquisition, or takeover (although the seller generally starts up again in the next CM with i.i.d. shocks, but we can add permanent shocks as in fn. 4). To explain the assumption of decreasing returns, without it, when two firms meet in the DM the one with higher ε should get all the capital. With decreasing returns, they *might* get all of it, but not necessarily. To rationalize decreasing returns, imagine entrepreneurs have some skill – e.g., managerial ability embodied in their presence at the firm – that constitutes a fixed factor. Note this is not inconsistent with a household owning shares in many firms, or a firm having many owners, since it concerns management, not ownership. Hence, we can allow households to diversify ε risk, but it is not actually important with quasi-linear utility, at least if $h \in [0, 1]$ is slack.

[7] Here q and k are one-for-one substitutable, but it is not difficult to assume, say, if firm 2 gets q from firm 1 it translates into only νq units of effective capital with $\nu < 1$ (e.g., when 2 buys 1's truck he has to repaint the logo on it). This captures a notion of specificity that may be interesting (e.g., for the choice of painting logos on trucks), but is relegated to future work.

Routine methods yield the planner's investment Euler equation

$$
\begin{aligned}
r_t + \delta = {} & (1-\alpha) \int_0^\infty F_1[k_{t+1}, h^*(k_{t+1}, \hat{\varepsilon}), \hat{\varepsilon}] dG(\hat{\varepsilon}) \\
& + \alpha \int_{\{\hat{\varepsilon} \geq \tilde{\varepsilon}\}} F_1\{k_{t+1} + q^*(\hat{\mathbf{s}}), h^*[k_{t+1} + q^*(\hat{\mathbf{s}})], \hat{\varepsilon}\} dG(\tilde{\varepsilon}) \, dG(\hat{\varepsilon}) \\
& + \alpha \int_{\{\hat{\varepsilon} < \tilde{\varepsilon}\}} F_1\{k_{t+1} - q^*(\tilde{\mathbf{s}}), h^*[k_{t+1} - q^*(\tilde{\mathbf{s}})], \hat{\varepsilon}\} dG(\tilde{\varepsilon}) \, dG(\hat{\varepsilon}) .
\end{aligned}
\tag{4}
$$

where r_t is given by $1 + r_t = u'(x) / \beta u'(x_{+1})$. In equilibrium this is a real interest rate, but for now $1 + r_t$ is simply notation for the marginal rate of substitution at t. This implies that, while (4) may look like a static condition, it is not, since out of steady state k_t affects x_t and hence r_t. In any case, the LHS of (4) is the marginal cost of investment due to discounting and depreciation, and the RHS is the marginal benefit taking into account the shocks plus reallocation.

Given these results for efficient employment, reallocation and investment, consumption x_t is given by the usual conditions. One can show the efficient exists uniquely and is summarized as follows:

Proposition 1 *Given k_0 and the time path of g_t, the solution to the planner problem is described by paths for $\left\langle \hat{k}_t^*, q_t^*(\cdot), h_t^*(\cdot), x_t^* \right\rangle$, where $k_t^* \in \mathbb{R}$, $q_t^* : \mathbf{s} \to \mathbb{R}$, $h_t^* : (k, \varepsilon) \to \mathbb{R}$, and $x_t^* \in \mathbb{R}$ satisfy (4), (2), (1) and the constraints in (3).*

When we say above, and below, that paths for the endogenous variables are bounded, we mean the usual transversality condition for capital from growth theory and, after adding money, later, the analogous condition for real balances (see Rocheteau and Wright 2013). We do not dwell on this, however, since the focus here is mainly on steady states.

3 Pure-Credit Equilibrium

While our main interest is in economies with credit frictions, perfect credit is a good benchmark. This means that the firm with higher ε in DM meetings gets q

units of capital in exchange for debt, a promise to deliver d units of numeraire in the next CM (quasi-linear utility implies there is no loss of generality to restricting attention to one-period debt, since agents in the CM are happy to pay off d, rather than roll it over, at least as long as $0 < h < 1$ is slack). Denote the CM and DM value functions by $W(a, k, \varepsilon)$ and $V(k, \varepsilon)$, where the CM state includes the agent's financial asset position, capital holdings and productivity, while the DM state includes just capital and productivity. In general, $a = z - d - T$ where z is real cash balances (which are 0 with perfect credit, but that changes below), d is debt brought into the CM from the previous DM, and T is a lump sum tax.

The CM problem is then

$$W(a,k,\varepsilon) = \max_{x,h,\hat{k}}\{u(x) - Ah + \beta\mathbb{E}_{\hat{\varepsilon}}V_{+1}(\hat{k},\hat{\varepsilon})\} \tag{5}$$

$$\text{st } x + \hat{k} = wh + a + \Pi(k,\varepsilon) + (1-\delta)k$$

$$\Pi(k,\varepsilon) = \max_{\tilde{h}}\{F(k,\tilde{h},\varepsilon) - w\tilde{h}\}, \tag{6}$$

where in terms of numeraire the price of capital is 1, the wage is w, $\Pi(k, \varepsilon)$ is profit income, and we omit t subscripts where the timing is obvious. From profit maximization, labor demand is

$$\tilde{h}(k,\varepsilon) = \arg\max_{\tilde{h}}\{F(k,\tilde{h},\varepsilon) - w\tilde{h}\}. \tag{7}$$

Of course $\tilde{h}$ also depends on w, but that is subsumed in the notation, to highlight the dependence on (k, ε).[8]

Using the constraints, we reduce (5) to

$$\begin{aligned} W(a,k,\varepsilon) &= \frac{A}{w}\left[\Pi(k,\varepsilon) + a + (1-\delta)k\right] + \max_x\left\{u(x) - \frac{A}{w}x\right\} \\ &\quad + \max_{\hat{k}}\left\{-\frac{A}{w}\hat{k} + \beta\mathbb{E}_{\hat{\varepsilon}}V(\hat{k},\hat{\varepsilon})\right\}. \end{aligned}$$

[8]Note that labor demand $\tilde{h}$ by a firm does not generally coincide with the supply h of its owner. Indeed, with hours traded in the frictionless CM, theory cannot pin down who works for whom. An interesting extension might be to combine frictional labor and capital markets; Berentsen et al. (2010) and Dong and Xiao (2018) already integrate microfounded monetary theory and a labor market like Pissarides (2000), but those models have no capital.

When nonnegativity constraints are slack, the FOC's are

$$x \quad : \quad \frac{A}{w} = u'(x) \tag{8}$$

$$\hat{k} \quad : \quad \frac{A}{w} = \beta \mathbb{E}_{\hat{\varepsilon}} V_1(\hat{k}, \hat{\varepsilon}) \tag{9}$$

plus the budget equation, while the relevant envelope conditions are

$$W_1(a, k, \varepsilon) = \frac{A}{w} \tag{10}$$

$$W_2(a, k, \varepsilon) = \frac{A}{w}\left[F_1(k, \tilde{h}, \varepsilon) + 1 - \delta\right]. \tag{11}$$

In the next result, part (i) follows directly from (9), and part (ii) from (10):[9]

Lemma 1 *Given $h \in (0,1)$: (i)the CM choice of $\hat{k}$ is the same for all agents, independent of their (a, k, ε); and (ii) W is linear in a.*

As in the planner problem, DM meetings are characterized by $\mathbf{s} = (k_b, \varepsilon_b, k_s, \varepsilon_s)$, and the buyer is the one with the higher ε. Then the trading surpluses are

$$\begin{aligned} S_b(\mathbf{s}) &= W[-d(\mathbf{s}), k_b + q(\mathbf{s}), \varepsilon_b] - W(k_b, \varepsilon_b) \\ S_s(\mathbf{s}) &= W[d(\mathbf{s}), k_s - q(\mathbf{s}), \varepsilon_s] - W(k_s, \varepsilon_s), \end{aligned}$$

where the buyer gets $q(\mathbf{s})$ in exchange for debt $d(\mathbf{s})$. By the envelope conditions, these reduce these to

$$\begin{aligned} S_b(\mathbf{s}) &= \frac{A}{w}\left\{\Pi[k_b + q(\mathbf{s}), \varepsilon_b] - \Pi(k_b, \varepsilon_b) + (1-\delta)\, q(\mathbf{s}) - d(\mathbf{s})\right\} \\ S_s(\mathbf{s}) &= \frac{A}{w}\left\{\Pi[k_s - q(\mathbf{s}), \varepsilon_s] - \Pi(k_s, \varepsilon_s) - (1-\delta)\, q(\mathbf{s}) + d(\mathbf{s})\right\}. \end{aligned}$$

Again the constraint $q(\mathbf{s}) \leq k_s$ may bind in some meetings, but standard conditions guarantee it does not.

[9] While quasi-linear utility is used here, the same results hold for any $U(x, 1-h)$ that is homogeneous of degree 1, as can be proved using the method in Wong (2016), or for any $U(x, 1-h)$ if we assume indivisible labor and use employment lotteries (see fn. 13).

In the interest of tractability, assume that both parties observe $\mathbf{s}$. Then any standard mechanism can be used to determine the terms of trade. For now we use Kalai's (1977) proportional bargaining solution.[10] If θ is buyers' bargaining power, the Kalai solution can be found by choosing (p, q) to maximize S_b subject to feasibility, plus $S_b(\mathbf{s}) = \theta S(\mathbf{s})$, where $S(\mathbf{s}) = S_b(\mathbf{s}) + S_s(\mathbf{s})$. With perfect credit $q = q^*(\mathbf{s})$ equates marginal products, as in the planner problem, and $S_b(\mathbf{s}) = \theta S(\mathbf{s})$ determines

$$\begin{aligned} d^*(\mathbf{s}) &= \theta \{\Pi(k_s, \varepsilon_s) - \Pi[k_s - q^*(\mathbf{s}), \varepsilon_s]\} + (1-\delta) q^*(\mathbf{s}) \\ &\quad + (1-\theta)\{\Pi[k_b + q^*(\mathbf{s}), \varepsilon_b] - \Pi(k_b, \varepsilon_b)\}. \end{aligned} \tag{12}$$

Recall that all agents have the same probability α of meeting a DM counterparty, drawn at random from the population, after $\hat{\varepsilon}$ is realized. So, before meetings occur, the expected payoff is

$$V(\hat{k}, \hat{\varepsilon}) = W(0, \hat{k}, \hat{\varepsilon}) + \alpha \int_{\hat{\varepsilon} > \tilde{\varepsilon}} S_b(\hat{\mathbf{s}})\, dG(\tilde{\varepsilon}) + \alpha \int_{\hat{\varepsilon} < \tilde{\varepsilon}} S_s(\tilde{\mathbf{s}})\, dG(\tilde{\varepsilon}). \tag{13}$$

The first term is the payoff from not trading in the DM, which is the continuation value in the next CM. The second is the surplus from buying capital in the DM, where in equilibrium $\hat{\mathbf{s}} = (\hat{k}, \hat{\varepsilon}, \hat{k}, \tilde{\varepsilon})$ and $\tilde{\mathbf{s}} = (\hat{k}, \tilde{\varepsilon}, \hat{k}, \hat{\varepsilon})$ because a firm is a buyer when it realizes $\hat{\varepsilon}$ and meets one with $\tilde{\varepsilon} < \hat{\varepsilon}$. The third term is the surplus from selling in the DM because $\hat{\varepsilon} < \tilde{\varepsilon}$. After reallocation, output y is given by the same equation as in the planner problem except $\tilde{h}$ replaces h^*. Then goods market clearing requires $x + g + \hat{k} = y + (1-\delta)k$, while labor market clearing can be ignored, by Walras' Law.

We now define equilibrium, where to conserve notation we do not carry around firm-specific labor demand, given by (7), only aggregate h. Then, continuing to omit t subscripts when obvious, we have this:

[10] As Aruoba et al. (2007) pointed out, Kalai has several advantages over generalized Nash bargaining in models with liquidity considerations, but with perfect credit they are the same.

Definition 1 *Given* k_0 *and time paths for* $\langle g, T\rangle$*, a pure-credit equilibrium is a list of paths for* $\left\langle \hat{k}, q(\cdot), p(\cdot), h, x, w \right\rangle$ *such that* $\forall t$*: (i)* $(x, h, \hat{k})$ *solves the CM maximization problem; (ii)* $p(\cdot)$ *and* $q(\cdot)$ *solve the DM bargaining problem; and (iii) markets clear.*

Sometimes we concentrate on a simpler notion:

Definition 2 *Given constant* $\langle g, T\rangle$*, a pure-credit steady state is a time-invariant list* $\left\langle \hat{k}, q(\cdot), p(\cdot), h, x, w \right\rangle$ *that satisfies the definition of equilibrium except for initial conditions.*

With perfect credit, reallocation is efficient in any meeting, but that does not mean investment is efficient. To discuss that, we need the capital Euler equation, which after routine algebra reduces to

$$\begin{aligned} r+\delta &= (1-\alpha)\int_0^\infty \Pi_1(\hat{k},\hat{\varepsilon})dG(\hat{\varepsilon}) \\ &\quad +\alpha\int_{\hat{\varepsilon}>\tilde{\varepsilon}}\left[\theta\Pi_1[\hat{k}+q(\hat{\mathbf{s}}),\tilde{\varepsilon}]+(1-\theta)\Pi_1(\hat{k},\hat{\varepsilon})\right]dG(\tilde{\varepsilon})dG(\hat{\varepsilon}) \\ &\quad +\alpha\int_{\hat{\varepsilon}<\tilde{\varepsilon}}\left[(1-\theta)\Pi_1[\hat{k}-q(\tilde{\mathbf{s}}),\hat{\varepsilon}]+\theta\Pi_1(\hat{k},\hat{\varepsilon})\right]dG(\tilde{\varepsilon})dG(\hat{\varepsilon}). \end{aligned} \tag{14}$$

By the envelope theorem, $\Pi_1(\cdot) = F_1(\cdot)$, making (14) look more like (4) from the planner problem, but the conditions are still different due to θ. In particular, comparing (14) to (4), the first line is the same, the second is the same iff $\theta = 1$, and the third is the same iff $\theta = 0$. So equilibrium is not generally efficient.

This is attributable to *holdup problems* in bargaining, about which we say more below, but here is the intuition. On the one hand, $\theta < 1$ increases individual demand for k in the primary market, relative to efficient investment, since buying in the secondary market is less attractive when sellers extract part of the surplus. On the other hand, $\theta > 0$ decreases demand for k in the primary market, since the option to sell it in the secondary market is less attractive when buyers extract part of the surplus. The former channel leads to overaccumulation of k, while the latter leads to underaccumulation, and as one should expect given Mortensen (1982) and Hosios (1990), there is a $\theta^* \in (0,1)$ that delivers efficiency. However, this θ^* is not constant out of n steady state, but depends on k_t. This is stated without proof as it is a special case of results derived below.

Proposition 2 *In pure-credit equilibrium, consumption, hours and reallocation are efficient conditional on investment, while investment is too high if $\theta < \theta^*$, too low if $\theta > \theta^*$ and efficient if $\theta = \theta^*$, where θ^* is not constant over time, but depends on k_t.*

4 Monetary Equilibrium

It is well known (see the surveys cited in fn. 2) that money is essential only when credit is imperfect. This requires imperfect commitment, so that we cannot trivially say agents honor debts, plus imperfect information, so that we cannot get them to honor debts by punishing those who renege. If sufficient information were available about those who renege, the threat of taking away future credit can dissuade opportunistic default, as in Kehoe and Levine (1983). We assume information is insufficient to support any unsecured credit (all we need is that it is insufficient to emulate perfect credit; see Gu et al. 2016).

Alternatively, one could punish defaulters by taking away profits, as in Holmstrom and Tirole (1998). To make this it interesting, such models typically assume only a fraction χ_1 of future profits are *pledgeable* – i.e., can be seized in the event of default. Additionally, agents can try to use current asset holdings to facilitate intertemporal exchange, as in Kiyotaki and Moore (1997). At the time of a DM transaction, our buyers hold k units of capital from the CM, and could in principle pledge a fraction χ_2 of that, plus a fraction χ_3 of newly-purchased capital q. We assume $\chi_1 = \chi_2 = 0$, although all we really need is that pledgeability is insufficient to emulate perfect credit (again see Gu et al. 2016). But with a nod to realism (e.g., Gavazza 2011a), we set $\chi_3 = 1$, so that newly purchased capital is fully pledgeable after depreciation.[11]

Securing credit with newly purchased q, or at least $(1-\delta)\,q$, is like getting a mortgage to buy a house. It is also equivalent to a rental agreement: a firm pays something up front to lease q, and returns $(1-\delta)\,q$ later. If this is not obvious, note that in any frictionless market returning $(1-\delta)\,q$ is equivalent to keeping it and settling debt with the same value. Still, $\chi_3 = 1$ is not enough to emulate perfect credit. Consider trying to use credit with only $(1-\delta)\,q$ as collateral: no deal can be done, because the capital is worth $(1-\delta)\,q$ to the seller even without using in production. Thus, buyers must provide a *cash down payment*, interpreted as *internal finance*, as discussed by Bernanke et al. (1999). Let m denote an agent's nominal balances and ϕ the price of money in terms of numeraire, so $z = \phi m$ is real balances. Then any DM meeting is characterized by $\mathbf{s} = (z_b, k_b, \varepsilon_b, z_s, k_s, \varepsilon_s)$, with a slight abuse of notation, since the z's show up. Given $\mathbf{s}$, as before $q(\mathbf{s})$ and $d(\mathbf{s})$ is debt, while $p(\mathbf{s})$ is now the down payment.

Let the money supply follow $M_{+1} = (1+\mu)\,M$, where μ is determined by policy, so the government's CM budget constraint becomes $g = T + \phi(M_{+1} - M)$.[12] The inflation rate is $\pi = \phi/\phi_{+1} - 1$. By the Fisher equation, $1 + \iota = (1+r)\,(1 + \phi/\phi_{+1})$ gives the return on an nominal bond that is illiquid – i.e., cannot be traded in the DM – just like $1 + r = u'(x)\,/\beta u'(x_{+1})$ gives the return on a real bond that is illiquid. As usual, these bonds can be priced whether or not they trade in equilibrium: $1 + \iota_t$ is simply the amount of cash in the CM at $t+1$ that makes you willing to give up a dollar in the CM at t, and $1 + r_t$ is the analog for numeraire. In steady state, $\pi = \mu$ and $1 + \iota = (1+\mu)\,/\beta$, so it is equivalent

[11] Having new capital more pledgeable than old can be motivated by saying, e.g., that it is difficult to punish defaulters by seizing their used stuff because we do not know where it is, or perhaps how good it is, as formalized in Li et al. (2013).

[12] It does not matter here if the new money goes to reduce the tax T or pay for government consumption g. Alternatively, we could introduce government bonds and inject money by open market operations, as in Rocheteau et al. (2018), which may be interesting, but may also be a dsitraction for present purposes.

to describe monetary policy by a choice of μ, π or ι. As is standard we impose $\mu > \beta - 1$, equivalent in steady state to $\iota > 0$; we also consider the limit $\mu \rightarrow \beta - 1$, or $\iota \rightarrow 0$, which is the Friedman rule.

The CM problem is similar to (5), except the budget equation is now

$$x + \hat{k} + (1+\pi)\hat{z} = wh + a + \Pi(k, \varepsilon) + (1-\delta)k, \tag{15}$$

where $1+\pi$ is the current price of real balances for the next DM. Applying similar methods to those used above, the key FOC's are

$$\hat{z} \; : \; \frac{A(1+\pi)}{w} = \beta \mathbb{E}_{\hat{\varepsilon}} V_1(\hat{z}, \hat{k}, \hat{\varepsilon}) \tag{16}$$

$$\hat{k} \; : \; \frac{A}{w} = \beta \mathbb{E}_{\hat{\varepsilon}} V_2(\hat{z}, \hat{k}, \hat{\varepsilon}), \tag{17}$$

while the envelope conditions are again (10)-(11). The generalization of Lemma 1 is:

Lemma 2 *Given $h \in (0,1)$: (i) the CM choice $(\hat{z}, \hat{k})$ is the same for all agents independent of individual (a, k, ε); (ii) W is linear in a.*

Moving to the DM, the trading surpluses are

$$\begin{aligned} S_b(\mathbf{s}) &= \frac{A}{w}\left\{\Pi[k_b + q(\mathbf{s}), \varepsilon_b] - \Pi(k_b, \varepsilon_b) + (1-\delta)\, q(\mathbf{s}) - p(\mathbf{s}) - d(\mathbf{s})\right\} \\ S_s(\mathbf{s}) &= \frac{A}{w}\left\{\Pi[k_s - q(\mathbf{s}), \varepsilon_s] - \Pi(k_s, \varepsilon_s) - (1-\delta)\, q(\mathbf{s}) + p(\mathbf{s}) + d(\mathbf{s})\right\}. \end{aligned}$$

There are now three constraints: $q(\mathbf{s}) \leq k_s$; $d(\mathbf{s}) \leq (1-\delta)\, q(\mathbf{s})$; and $p(\mathbf{s}) \leq z_b$. As before, standard conditions make the first one slack. For the second, while the buyer may as well use all the credit he can before tapping his cash, he still needs some cash as a down payment. We formalize this as follows:

Lemma 3 *In monetary equilibrium, in all DM trade $d(\mathbf{s}) = (1-\delta)\, q(\mathbf{s})$ and $p(\mathbf{s}) > 0$.*

Based on these observations the previous expressions simplify to

$$S_b(\mathbf{s}) = \frac{A}{w}\{\Pi[k_b + q(\hat{\mathbf{s}}), \varepsilon_b] - \Pi(k_b, \varepsilon_b) - \hat{p}(\mathbf{s})\} \tag{18}$$

$$S_s(\mathbf{s}) = \frac{A}{w}\{\Pi[k_s - \hat{q}(\mathbf{s}), \varepsilon_s] - \Pi(k_s, \varepsilon_s) + \hat{p}(\mathbf{s})\}. \tag{19}$$

Notice $(1-\delta)\, q(\mathbf{s})$ and $d(\mathbf{s})$ cancel in (18) and (19), although debt is still relevant, because it allows agents to economize on cash. Now, as regards the third constraint, $p(\mathbf{s}) \leq z_b$, for at least some $\mathbf{s}$ it must bind because. Intuitively, this is because $\iota > 0$ makes cash a poor saving vehicle, so agents do not carry more than they would ever spend. Then as in Gu and Wright (2016) and Zhu (2018), one can show the following: (i) if $p \leq z_b$ is slack, $q^*(\mathbf{s})$ is the same as perfect credit, and the mechanism determines $p^*(\mathbf{s})$; (ii) if $p \leq z_b$ binds, $p(\mathbf{s}) = z_b$ and the mechanism determines $q(\mathbf{s}) < q^*(\mathbf{s})$. So there is a set $\mathcal{B}$ such that $p \leq z_b$ binds iff $\mathbf{s} \in \mathcal{B}$, where prob$(\mathbf{s} \in \mathcal{B}) > 0$.

With Kalai bargaining, e.g., $\mathbf{s} \notin \mathcal{B}$ implies $q = q^*(\mathbf{s})$ and

$$p^*(\mathbf{s}) = (1-\theta)\{\Pi[k_b + q^*(\mathbf{s}), \varepsilon_b] - \Pi(k_b, \varepsilon_b)\} + \theta\{\Pi(k_s, \varepsilon_s) - \Pi[k_s - q^*(\mathbf{s}), \varepsilon_s]\};$$

and $\mathbf{s} \in \mathcal{B}$ implies $p(\mathbf{s}) = z_b$ and $q = q(\mathbf{s})$ where

$$z_b = (1-\theta)\{\Pi[k_b + q(\mathbf{s}), \varepsilon_b] - \Pi(k_b, \varepsilon_b)\} + \theta\{\Pi(k_s, \varepsilon_s) - \Pi[k_s - q(\mathbf{s}), \varepsilon_s]\}.$$

Then output y is as above. Goods market clearing is also the same, money market clearing is simply $m = M$, and labor market clearing is again ignored by Walras' Law. Given all agents start with (z_0, k_0), we define monetary equilibrium as an outcome with $z > 0$.[13]

Let us specify monetary policy by the growth rate μ, although again, for steady state that is equivalent to setting π or ι. Then we have this:

[13] There also exists a nonmonetary equilibrium, which reduces to the standard growth model. Indeed, if we eliminate idiosyncratic and add aggregate productivity shocks, the nonmonetary equilibrium replicates exactly Hansen's (1985) real business cycle model. The only detail is that he does not start with $u(x) - Ah$; he derives it from general utility $U(x, h)$ by imposing indivisible labor, $h \in \{0, 1\}$, and incorporating employment lotteries à la Rogerson (1988). The same method works here: assuming indivisible labor and lotteries, agents with any $U(c, h)$ act *as if* their utility functions were $u(x) - Ah$ (e.g., see Rocheteau et al. 2008).

Definition 3 *Given (z_0, k_0) and paths for $\langle \mu, g, T \rangle$, monetary equilibrium is a list of paths for $\left\langle \hat{k}, q(\cdot), p(\cdot), h, x, \hat{z}, w \right\rangle$ such that for $\forall t$: (i) $(x, h, \hat{z}, \hat{k})$ solves the CM maximization problem; (ii) $p(\cdot)$ and $q(\cdot)$ solve the DM bargaining problem; and (iii) markets clear.*

While there can be dynamics in monetary economies based on beliefs, as well as transitional dynamics, we usually concentrate on a simpler concept:

Definition 4 *Given constant $\langle \mu, g, T \rangle$, monetary steady state is a time-invariant list $\left\langle \hat{k}, q(\cdot), p(\cdot), h, x, \hat{z}, w \right\rangle$ that satisfies the definition of equilibrium except for initial conditions.*

To get the Euler equations for money and capital, we need the derivatives of $p(\cdot)$ and $q(\cdot)$. Supposing that $\mathbf{s} \in \mathcal{B}$ (the other case is similar), we have $\partial p/\partial z_b = 1$, $\partial p/\partial k_b = \partial p/\partial k_s = 0$ and

$$
\begin{aligned}
\frac{\partial q}{\partial z_b} &= \frac{1}{D(\mathbf{s})} && (20) \\
\frac{\partial q}{\partial k_b} &= (1-\theta) \frac{\Pi_1(k_b, \varepsilon_b) - \Pi_1[k_b + q(\mathbf{s}), \varepsilon_b]}{D(\mathbf{s})} && (21) \\
\frac{\partial q}{\partial k_s} &= \theta \frac{\Pi_1[k_s - q(\mathbf{s}), \varepsilon_s] - \Pi_1(k_s, \varepsilon_s)}{D(\mathbf{s})}, && (22)
\end{aligned}
$$

where

$$D(\mathbf{s}) \equiv (1-\theta)\, \Pi_1[k_b + q(\mathbf{s}), \varepsilon_b] + \theta \Pi_1[k_s - q(\mathbf{s}), \varepsilon_s]. \tag{23}$$

Evaluating these at $\hat{\mathbf{s}} = (\hat{z}, \hat{k}, \hat{\varepsilon}, \hat{z}, \hat{k}, \tilde{\varepsilon})$, inserting them into the derivatives of $V(\cdot)$ wrt $(\hat{z}, \hat{k})$, and inserting those into the FOC's (16)-(17), we get the Euler equations.

For money, the result is

$$\frac{1+\pi}{w} = \frac{\beta}{w_{+1}} \left[1 + \alpha \int_{\{\hat{\varepsilon} > \tilde{\varepsilon}\}} \Lambda(\hat{\mathbf{s}})\, dG(\tilde{\varepsilon}) dG(\hat{\varepsilon}) \right], \tag{24}$$

where

$$\Lambda(\hat{\mathbf{s}}) \equiv \theta \frac{\Pi_1[\hat{k} + q(\hat{\mathbf{s}}), \hat{\varepsilon}] - \Pi_1[\hat{k} - q(\hat{\mathbf{s}}), \tilde{\varepsilon}]}{D(\hat{\mathbf{s}})}. \tag{25}$$

Observe that $w_{+1}/\beta w = u'(x)/\beta u'(x_{+1}) = 1+r$, and recall $(1+\pi)(1+r) = 1+\iota$ (the Fisher equation). Then (24) becomes

$$\iota = \alpha \int_{\{\hat{\varepsilon} \geq \tilde{\varepsilon}\}} \Lambda(\hat{\mathbf{s}})\, dG(\tilde{\varepsilon}) dG(\hat{\varepsilon}), \tag{26}$$

where the LHS is the nominal interest rate, or the marginal cost of holding cash, and the RHS is the benefit; in fact, $\Lambda(\hat{\mathbf{s}})$ is the Lagrange multiplier on $p \leq z_b$, which New Monetarists call the *liquidity premium*. Hence, given $\hat{k}$, reallocation is efficient iff $\iota = 0$, while $\iota > 0$ a wedge in the sense of a tax on DM trade.

Similarly, for capital, the Euler equation is

$$\begin{aligned} r + \delta \;\; = \;\; & (1-\alpha) \int_0^\infty \Pi_1(\hat{k}, \hat{\varepsilon}) dG(\hat{\varepsilon}) \\ & + \alpha \int_{\{\hat{\varepsilon} \geq \tilde{\varepsilon}\}} \Pi_1[\hat{k} + q(\hat{\mathbf{s}}), \hat{\varepsilon}] \Omega(\hat{\mathbf{s}})\, dG(\tilde{\varepsilon}) dG(\hat{\varepsilon}) \\ & + \alpha \int_{\{\hat{\varepsilon} < \tilde{\varepsilon}\}} \Pi_1[\hat{k} - q(\tilde{\mathbf{s}}), \hat{\varepsilon}] \Gamma(\tilde{\mathbf{s}})\, dG(\tilde{\varepsilon}) dG(\hat{\varepsilon}), \end{aligned} \tag{27}$$

where $\hat{\mathbf{s}} = (\hat{z}, \hat{k}, \hat{\varepsilon}, \hat{z}, \hat{k}, \tilde{\varepsilon})$, $\tilde{\mathbf{s}} = (\hat{z}, \hat{k}, \tilde{\varepsilon}, \hat{z}, \hat{k}, \hat{\varepsilon})$ interchange $\hat{\varepsilon}$ and $\tilde{\varepsilon}$, while

$$\Omega(\hat{\mathbf{s}}) \equiv \frac{(1-\theta)\Pi_1(\hat{k}, \hat{\varepsilon}) + \theta \Pi_1[\hat{k} - q(\hat{\mathbf{s}}), \tilde{\varepsilon}]}{D(\hat{\mathbf{s}})} \tag{28}$$

$$\Gamma(\tilde{\mathbf{s}}) \equiv \frac{(1-\theta)\Pi_1[\hat{k} + q(\tilde{\mathbf{s}}), \hat{\varepsilon}] + \theta \Pi_1(\hat{k}, \tilde{\varepsilon})}{D(\tilde{\mathbf{s}})}. \tag{29}$$

Notice $\theta < 1$ implies $\Omega(\hat{\mathbf{s}}) > 1$, which raises CM demand for k, because buying it in the DM is less attractive when sellers extract more surplus; and $\theta > 0$ implies $\Gamma(\tilde{\mathbf{s}}) < 1$, which lowers CM demand for k, because the option to sell in the DM is less attractive when buyers extract more surplus.

From (14) and (27), k is efficient at $\Omega(\hat{\mathbf{s}}) = \Gamma(\tilde{\mathbf{s}}) = 1$, and $\theta > 0$ implies under-investment while $\theta < 1$ implies over investment. It remains true that there exists $\theta^* \in (0,1)$ such that, as long as the liquidity wedge is shut down by setting $\iota = 0$, we get the first best at $\theta = \theta^*$. We cannot get the first best at $\iota > 0$, but there is a θ^{**} such equilibrium attains the second best. Further, monetary policy affects investment here through interesting and novel channels. To explain, notice higher ι increases the demand for k in the CM because buying it in the DM is less attractive when liquidity is more expensive, reminiscent of the Mundell-Tobin effect of inflation, even if our microfoundations are different. But there is another effect, whereby higher ι decreases demand for z, and that reduces CM investment, because the option value of selling k in the DM is less attractive when there is less "cash in the market," reminiscent of Keynesian suggestions that lowering nominal interest rates stimulates investment, although again the microfoundations are quite different. Below we ask, which effect dominates?

First, in terms of the literature, Kurman and Rabinovitz (2018) and references therein have k holdup problems, but not m holdup problems, since they have no money. The monetary papers in the surveys cited in fn. 2 have m holdup problems but not k holdup problems, with a few exceptions, like Aruoba et al. (2011), but there agents trade consumption in the DM, not capital, so our investment effect are absent. Wright et al. (2017) has m and k holdup problems, but the implications are very different, because there some agents bring k but not m to the DM while others bring m but not k, while here all agents bring both. That makes the earlier setup is more like labor models by Masters (1998, 2011) or Acemoglu and Shimer (1999), where firms invest only in physical capital and workers only in human capital.

This is not a minor technicality – it makes a major difference. When some agents bring k and others bring m there is no $\theta = \theta^*$ that attains the first best even at $\iota = 0$. Also, in general the k holdup problem tends to reduce investment, while the m holdup problem tends to reduce real balances and hence reallocation, but there is a new effect here, when all agents bring k and m to the DM, because the m holdup problem tends to increase investment. This novel effect is missing in models where agents bring k or m but not both.[14]

[14] In terms of a different but related literature, our result is similar to models of household production (e.g., Burdett et al. 2016) where inflation is a tax on monetary exchange, and hence on market relative to home activity. In those models, higher ι gives households more incentive

In the next result, part (i) follows immediately from noticing that monetary exchange replicates perfect credit at the Friedman rule: the same θ^* that gives the first best with perfect credit gives the first best with money when $\iota = 0$. Then part (ii) follows directly from (26), which says reallocation is not efficient away from the the Friedman rule. We say more about these issues below; for now we catalogue the observations as follows:

Proposition 3 *(i) In monetary equilibrium with $\iota = 0$, consumption, hours and reallocation are efficient conditional on investment, while investment is too high if $\theta < \theta^*$, too low if $\theta > \theta^*$, and efficient if $\theta = \theta^*$, where θ^* depends on k_t. (ii) With $\iota > 0$ monetary equilibrium is not efficient.*

5 A Convenient Parameterization

The general model can be studied quantitatively, in steady state or otherwise, with or without i.i.d. shocks. Here we consider a simpler specification that yields sharp analytic results as a way to develop and communicate salient economic ideas. Thus, assume $\varepsilon \in \{\varepsilon_H, \varepsilon_L\}$ with $\varepsilon_H > \varepsilon_L$, where prob$(\varepsilon_L) = \gamma_L$ and prob$(\varepsilon_H) = \gamma_H = 1 - \gamma_L$. This is convenient as it implies $p \leq z_b$ binds in every DM trade – it must bind in some meetings, and now there is only one relevant kind of meeting, where a firm with ε_H meets one with ε_L. Also, now the bargaining solution is a number q, rather than a function $q(\mathbf{s})$, and that lets us easily depict equilibrium in (k, q) space. It also means $\alpha_H = \alpha\gamma_L$ and $\alpha_L = \alpha\gamma_H$.[15] Here we

to do their own cooking, cleaning, child care etc., instead of getting similar goods and services on the market. By analogy, here higher ι gives firms more incentive to accumulate their own capital in the primary market, instead of getting it on the secondary market.

[15] One can generalize this by letting $\alpha_L = \varkappa(\gamma_H, \gamma_L)/\gamma_L$ and $\alpha_H = \varkappa(\gamma_H, \gamma_L)/\gamma_H$ for any matching function $\varkappa(\cdot)$ satisfying the usual monotonicity and concavity conditions. Given $\chi(\cdot)$ displays constant returns, we can write $\alpha_L = \varkappa(n, 1) = \alpha(n)$ and $\alpha_H = \varkappa(n, 1)/n = \alpha(n)/n$, where $n = \gamma_H/\gamma_L$ is called *market tightness*, while $\alpha(n)$ is increasing and concave with $\alpha(n) \leq \min\{1, n\}$. Is is also sometimes useful to impose $\lim_{n\to 0} \alpha'(n) = \infty$. Of course the endogenous meeting probabilities are more interesting when n is not fixed, as when participation in the DM is costly, or as in the model in Section .

also use technology $F(k,h,\varepsilon) = \varepsilon f(k) + h$, which may not be great in quantitative work, but is very useful for deriving analytic results, basically because it implies $w = 1$, which simplifies a few calculations.

To begin, consider a planner problem like (3), except output is

$$y = Cf(k) + \xi\varepsilon_H f(k+q) + \xi\varepsilon_L f(k-q) + h,$$

where $\xi = \gamma_H\alpha_H = \gamma_L\alpha_L$ is the measure of firms that trade in the DM, while $C = \gamma_L(1-\alpha_L)\varepsilon_L + \gamma_H(1-\alpha_H)\varepsilon_H$ is average productivity for the rest (while we already solved the general problem, it is worth revisiting it in this special case). The FOC's for q and k are

$$0 = \varepsilon_H f'(k+q) - \varepsilon_L f'(k-q) \quad (30)$$

$$r + \delta = Cf'(k) + \xi\varepsilon_H f'(k+q) + \xi\varepsilon_L f'(k-q) \quad (31)$$

(which are considerably simpler than the general case). Now impose steady state, so that $r = 1/\beta - 1$.[16]

Condition (30) defines $q = Q(k)$ and (31) defines $k = K(q)$, two (single-valued) functions. We call $k = K(q)$ the IS curve, a standard name for the investment Euler equation, and call $q = Q(k)$ the CR curve, for capital reallocation. Efficiency obtains in (k,q) space when they cross. Their slopes are given

[16]Different from the general model, (31) is basically a static condition: while r is still given by $1 + r = u'(x)/\beta u'(x_{+1})$, with $w = 1$ we get $u'(x) = A$ in and out of steady state. This means the economy can jump to steady state in one period *unless* $h \in [0,1]$ binds. To see what this means, consider the case with no DM – i.e., the standard growth model with utility and production functions linear in h. It has a unique steady state $\bar{k}$ (notice 0 is not a steady state since y can be produced with h even at $k = 0$). At $\bar{k}$, x solves $u'(\bar{x}) = A$ and h solves $\bar{h} = \bar{x} + \delta\bar{k} - f(\bar{k})$ as long as $\bar{h} \le 1$. Now suppose the initial k_0 is below $\bar{k}$, but close; then we jump to $\bar{k}$ in one period by setting $h_0 = \bar{x} + \bar{k} - f(k_0) - (1-\delta)k_0$. Now suppose k_0 is so low that $\bar{x} + \bar{k} - f(k_0) - (1-\delta)k_0 > 1$, and $h \le 1$ initially binds; then the transition has $h_t = 1$, with x_t and k_t determined in the obvious way, for $t = 1, 2, \ldots$, until we reach k such that $h = \bar{x} + \bar{k} - f(k) - (1-\delta)k \le 1$, when we jump to $\bar{k}$.

by

$$\begin{aligned}\frac{\partial q}{\partial k}_{|CR} &= \frac{\Phi(k,q)}{\varepsilon_H f''(k+q)+\varepsilon_L f''(k-q)}\\ \frac{\partial q}{\partial k}_{|IS} &= \frac{\varepsilon_H f''(k+q)+\varepsilon_L f''(k-q)+Cf''(k)/\xi}{\Phi(k,q)}\end{aligned}$$

where we define

$$\begin{aligned}\Phi(k,q) &\equiv \varepsilon_L f''(k-q)-\varepsilon_H f''(k+q)\\ &= f'(k+q)f''(k-q)-f'(k-q)f''(k+q),\end{aligned}$$

with the second line following from (30). Hence, when they cross, IS and CR both slope down if $\Phi(k,q)>0$ and up $\Phi(k,q)<0$.

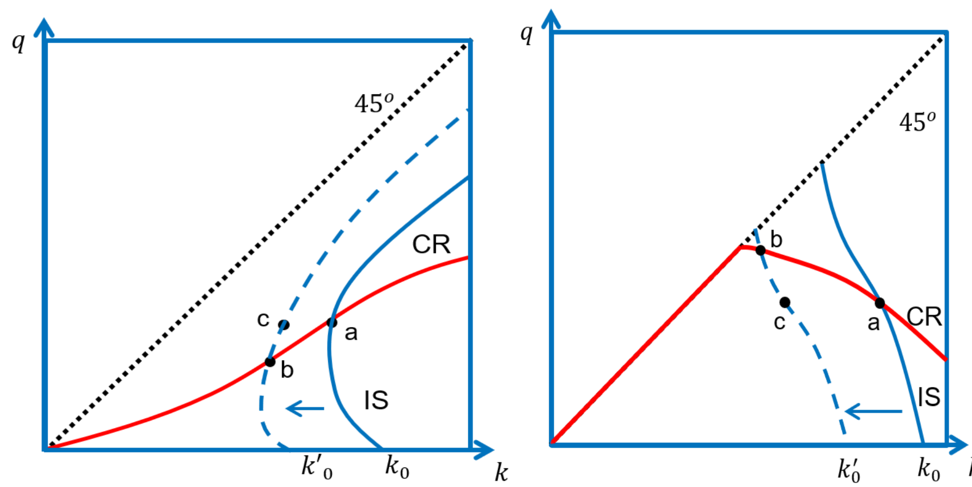


Figure 2: IS and CR for planner with $\Phi<0$ (left) or $\Phi>0$ (right)

Both cases are possible, as shown in Figure 2.[17] The left panel has $\Phi<0$ so at the efficient outcome, where they cross, both curves slope up; the right panel has $\Phi>0$ so at the efficient outcome, both curves slope down. As an aside, notice

[17]For an explicit example that looks like get the left panel, use $f(k)=k^{1/3}$, $r=0.01$, $\delta=0.02$, $\gamma_H=\gamma_L=0.5$, $\varepsilon_L=0.6$, $\varepsilon_H=1$, $\theta=0.7$ and $\tau=0.3$. For an example that looks like the right panel, use $f(k)=k-0.4k^2+0.01k^3$, which is increasing and concave over the relevant range, if not globally, $r=\delta=0.1$, $\gamma_H=\gamma_L=0.5$, $\varepsilon_L=0.3$, $\varepsilon_H=1$, $\theta=0.7$ and $\tau=0$.

in the right panel that when k is low the CR curve coincides with the 45^o line, $q = k$. For the IS curves depicted in the graph, drawn for different parameters, at the efficient outcome we are below the 45^o line, so $q < k$. However, it is easy to change parameters – e.g., raise $r + \delta$ to shift IS further left – and then $q = k$ at efficient outcome. In that case efficiency implies a corner solution for capital reallocation: the higher productivity firm "buys out" the lower productivity firm, which as mentioned above can be interpreted as a merger or acquisition.

Another kind of corner solution is also possible – one where the efficient outcome has $k = 0$, so there is no investment, and production ensues using only h. Since that is not very interesting, we rule it out by imposing $r + \delta < (C + \xi\varepsilon_H + \xi\varepsilon_L) f'(0)$. Then we have this result:

Proposition 4 *With our convenient parameterization, the solution to the planner problem has a unique steady state, $k > 0$ and $q > 0$.*

Proof: The statement is equivalent to saying the IS and CR curves in Figure 2 cross uniquely in (k, q) space at $k > 0$ and $q > 0$. To establish that, first, let $\Delta = (\partial q/\partial K)_{|IS} - (\partial q/\partial K)_{|CR}$ and notice

$$\Delta = \frac{\varepsilon_H f''(k+q) + \varepsilon_L f''(k-q) + Cf''(k)/\xi}{\Phi(k,q)} - \frac{\Phi(k,q)}{\varepsilon_H f''(k+q) + \varepsilon_L f''(k-q)}.$$

As regards uniqueness, if the curves cross suppose $\Phi(k,q) < 0$ at a point where they do. Then, letting $\approx$ indicate both sides take the same sign, we have

$$\begin{aligned}
\Delta &\approx [\varepsilon_H f''(k+q) + \varepsilon_L f''(k-q)]^2 \\
&\quad + [\varepsilon_H f''(k+q) + \varepsilon_L f''(k-q)] Cf''(k)/\xi - \Phi(k,q)^2 \\
&> [\varepsilon_H f''(k+q) + \varepsilon_L f''(k-q)]^2 - \Phi(k,q)^2.
\end{aligned}$$

Since the RHS can be shown to be positive, in this case IS is steeper than CR when they cross.

Next, suppose that if $\Phi(k,q) > 0$ at where the curves cross. Then

$$\begin{aligned}
\Delta &\approx -\left[\varepsilon_H f''(k+q) + \varepsilon_L f''(k-q)\right]^2 \\
&\quad -[\varepsilon_H f''(k+q) + \varepsilon_L f''(k-q)] C f''(k)/\xi + \Phi(k,q)^2 \\
&< -\left[\varepsilon_H f''(k+q) + \varepsilon_L f''(k-q)\right]^2 + \Phi(k,q)^2.
\end{aligned}$$

Since the RHS can be shown to be negative, IS is again steeper than CR when they cross. Because the functions $q = Q(k)$ and $k = K(q)$ defining the curves are single-valued, this implies they cannot cross more than once.

To show they must cross, first note that CR satisfies $Q(0) = 0$ and $0 < Q(k) \le k \; \forall k > 0$, and that as $k \to \infty$, $k - Q(k) \to \infty$, because $\varepsilon_H f'[k + Q(k)] > \varepsilon_H f'(k) \to 0$, implying $\varepsilon_L f'[k - Q(k)] \to 0$. Similarly, the IS curve satisfies $K(0) > 0$ and $K(q) - q \to c < \infty$ as $q \to \infty$. Now having the curves cross is equivalent to finding a solution to

$$Q \circ K(q) - q = 0, \tag{32}$$

where $\circ$ denotes the composite of two functions. Notice $Q \circ K(0) > 0$, and as $q \to \infty$

$$Q \circ K(q) - q = Q \circ K(q) - K(q) + K(q) - q \to c - \infty = -\infty.$$

Hence, there exists $\tilde{q} > 0$ solving (32), and the curves cross at $q = \tilde{q}$ and $k = \tilde{k} = K(\tilde{q})$. ■

While it is not needed in general, sometimes it is useful to impose

$$\Phi(k,q) < 0, \qquad \text{(Condition F)}$$

so that IS and CR slope up when they cross. This always holds for $f(k) = k^\eta$ with $\eta \in (0,1)$, e.g., so Condition F is not too stringent. Moreover, increasing $r + \delta$ shifts IS left and thus decreases k, and decreases q when IS and CR slope up, as shown in Figure 2 (it may seem natural for k and q to move in the same direction, and Condition F tells us when that happens, but there is no problem in theory if they move in opposite directions).

Now consider monetary equilibrium. For this, we also introduce a proportional tax τ on capital income, to investigate interactions between monetary and fiscal policy. Then the CM budget equation becomes

$$x + (1 + \pi)\hat{z} = h + a + (1 - \tau)\,\varepsilon f(k) + (1 - \delta)\,k - \hat{k} - T.$$

Also, as there is only one relevant kind of meeting in the DM, and there $p \leq z_b$ must bind, the bargaining solution becomes

$$\frac{z}{1-\tau} = (1-\theta)\,\varepsilon_H[f(k+q) - f(k)] + \theta\varepsilon_L[f(k) - f(k-q)], \tag{33}$$

where we note the appearance of τ on the LHS. Then the Euler equations become

$$\iota = \xi\Lambda(k,q) \tag{34}$$

$$\frac{r+\delta}{1-\tau} = Cf'(k) + \xi\varepsilon_H f'(k+q)\Omega(k,q) + \xi\varepsilon_L f'(k-q)\Gamma(k,q) \tag{35}$$

where $\Lambda(k,q)$, $\Omega(k,q)$ and $\Gamma(k,q)$ are simplified versions of the wedges in Section 4, and here we write their arguments as (k,q) rather than $\mathbf{s}$, again because there is only one relevant kind of DM meeting.

As in the planner's problem, (34) and (35) define the CR and IS curves, intersections of which yield (k,q), from which we get z, x, h as before. However, in this economy the CR is nothing more nor less than the LM curve from traditional Keynesian models – so let's call it that, even though we still plot it in (k,q) space, different from textbook presentations, because k and q capture our two main objects of interest: accumulation and reallocation.

While the method for characterizing monetary equilibrium is similar to the planner's problem, the results are different. First, monetary equilibrium exists iff ι is below a threshold $\bar{\iota}$, so if the inflation tax is too high the secondary market shuts down and all capital is acquired in the primary market. Second, uniqueness is more delicate due to complementarities – if there is more cash in the market you may want to bring more capital, and vice versa – but it can be established under some conditions. The proof of the following is in the Appendix.

Proposition 5 *With our convenient parameterization, monetary steady state exists iff*

$$\iota < \bar{\iota} \equiv \frac{\alpha_H \gamma_H \theta (\varepsilon_H - \varepsilon_L)}{(1-\theta)\varepsilon_H + \theta \varepsilon_L}.$$

It is unique if either θ is not too small or ι is not too big.

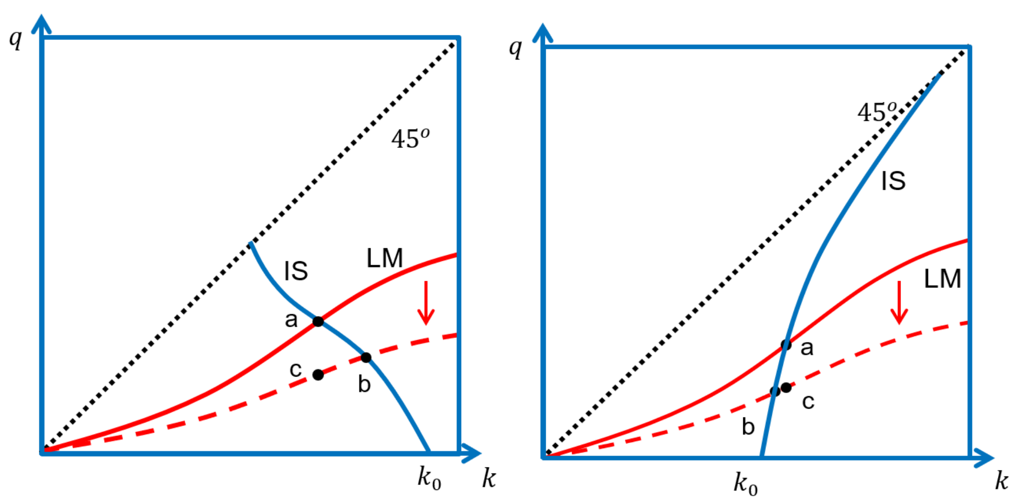
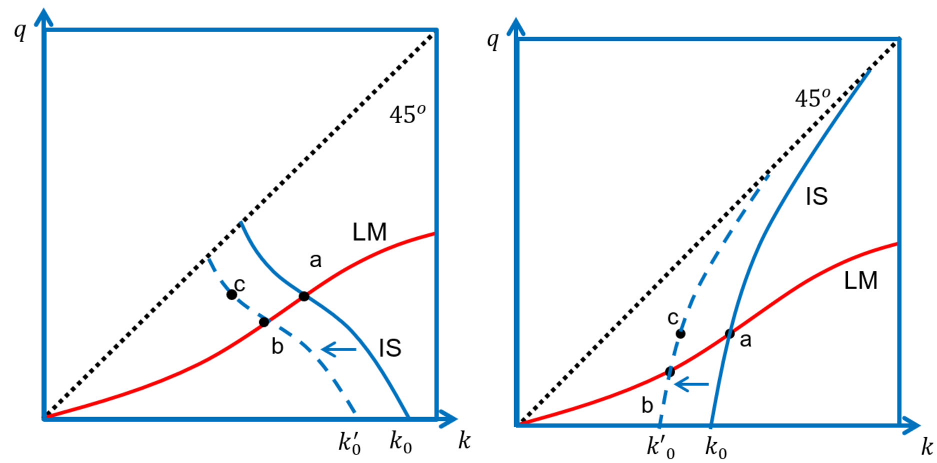


Figure 3: IS and LM, with monetary policy increasing ι

The results are illustrated in Figure 3. Similar to CR from the planner's problem, the LM curve starts at $(0,0)$, lies below the 45^0 line, and while it is not monotone increasing in general, it is under Condition F. Different from the planner's problem, even under Condition F the IS curve can be increasing, decreasing, or nonmonotone. This is related to the discussion in Section 4: first, there is less need to bring k from the CM when it is easier to get it in the DM, and that tends to make IS decreasing, as in the left panels of Figure 3; second, higher q means selling capital in the DM is more lucrative, and that tends to make IS increasing, as in the right panel. On net, if θ is big the first effect dominates and IS slopes down, while if θ is small the second effect dominates and IS slopes up.

Also shown in Figure 3 are the effects of monetary policy. An increase in ι does not affect IS, but rotates LM clockwise about the origin until ι hits $\bar{\iota}$, at which point LM hits the horizontal axis and monetary equilibrium breaks down. When monetary equilibrium exists, as ι increases steady state moves from a to b in the graph, with q decreasing and k decreasing (increasing) as the IS slopes up (down). So at least for parameters implying IS slopes up, lower nominal interest (or inflation or money growth) rates increase capital investment. This is consistent with Keynesian doctrine but the logic is quite different: here lower ι reduces the cost of liquidity, which facilitates trade in the secondary market, and at least for low θ that raises investment in the primary market because it raises the option value of selling it in the secondary market. To see the multipliers at work, observe that an increase in ι would move us from a to c if k were fixed, but since k in fact reacts, we go to b, attenuating the fall in q in the left panel and accentuating it in the right.



Case 1: High θ Case 2: Low θ

Figure 4: IS and LM, with fiscal policy increasing τ

Figure 4 shows the effects of fiscal policy captured by an increase in the profit tax τ. This shifts IS left but does not affect LM, so q and k both decrease, regardless of whether IS slopes up or down. To see the fiscal multipliers at work, in this case, observe that an increase in τ would move us from a to c if q were fixed, but since q in fact reacts, we instead move to b, which attenuates the fall in k in the left panel and accentuates it in the right panel. Hence, fiscal as well as monetary policy can be analyzed conveniently and clearly using simple graphs, similar to those used in common undergraduate macro courses, although with different microfoundations.[18]

If $\iota = \tau = 0$ then k is efficient in monetary equilibrium iff $\theta = \theta^*$ (where θ^* is

[18] As regards other parameters, e.g., related to productivity, at least for θ not too small we can show (see the Online Appendix) the following: Let $F^j(k, h) = B\varepsilon_j f(k) + h$, and note that output y moves in the same direction as k, since in steady state $x + \delta k = y$, where $x = u'^{-1}(A)$. Hence, higher k means higher y, or better economic times. Then k goes up with B, ε_H, ε_L and γ_H. Under Condition F, q also goes up with B and ε_H, but the dependence on ε_L and γ_H is ambiguous. Of course, these effects, like those in the text, assume a special technology that makes w fixed; a more general specification would mean more general equilibrium complications, but the forces identified are clearly still relevant.

the same as with perfect credit). In this specification that reduces to

$$\theta^* = \frac{\varepsilon_L \left[f'(k^* - q^*) - f'(k^*) \right]}{(\varepsilon_H - \varepsilon_L) f'(k^*)} \tag{36}$$

where (k^*, q^*) comes from the planner problem. If $\theta \neq \theta^*$ then efficiency still obtains at $\iota = 0$ if we set $\tau = \tau^*$, where

$$1 - \tau^* = \frac{r + \delta}{\xi \varepsilon_H f'(k^* + q^*) + (1 - \theta \alpha_H) \gamma_H \varepsilon_H f'(k^*) + [1 - (1 - \theta) \alpha_L] \gamma_L \varepsilon_L f'(k^*)}.$$

One can check that τ^* is decreasing in θ, which means we need a subsidy $\tau^* < 0$ if $\theta > \theta^*$ and a tax $\tau^* > 0$ if $\theta > \theta^*$. We summarize as follows:

Proposition 6 *Consider our convenient parameterization with $\iota = 0$. Then optimal fiscal policy in monetary steady state is $\tau = \tau^*$, where: $\tau^* = 0$ if $\theta = \theta^*$, $\tau^* < 0$ if $\theta > \theta^*$, and $\tau^* > 0$ if $\theta < \theta^*$. In each case we achieve the first best (k^*, q^*).*

Given these results on fiscal policy in monetary equilibrium when $\iota = 0$ (equivalently, in pure-credit equilibrium), consider monetary policy when $\tau = 0$. The Appendix proves the following results, which say that the Friedman rule may a good idea when θ is close to θ^*, but not when θ is less close to θ^*.

Proposition 7 *Consider our convenient parameterization with $\tau = 0$. Then there exist θ_L and θ_H, with $0 < \theta_L \leq \theta^* \leq \theta_H < 1$, such that optimal monetary policy is $\iota^* > 0$ for $\theta > \theta_H$ and for $\theta < \theta_L$, but in neither case do we achieve (k^*, q^*). For $f(k) = k^\eta$, we can solve explicitly for*

$$\begin{aligned} \theta_L &= \theta^* = \frac{\varepsilon_H - \omega^{1-\eta}}{\varepsilon_H - \varepsilon_L} \\ \theta_H &= \frac{\varepsilon_H - \varepsilon_L + \omega^{1-\eta} - \sqrt{\left[\varepsilon_H - \varepsilon_L + \omega^{1-\eta}\right]^2 - 2\frac{\omega^{1-\eta}\varepsilon_H^{\frac{1}{1-\eta}}(\varepsilon_H - \varepsilon_L)}{\omega}}}{2(\varepsilon_H - \varepsilon_L)} \end{aligned}$$

where $\omega = \frac{1}{2}\left(\varepsilon_H^{\frac{1}{1-\eta}} + \varepsilon_L^{\frac{1}{1-\eta}}\right)$. For the same $f(k)$, as a special case of the results, $\iota^* > 0$ is optimal for $\theta \notin (\theta_L, \theta_H)$, and $\iota^* = 0$ is at least locally optimal for $\theta \in (\theta_L, \theta_H)$; it is globally optimal if welfare is concave in ι.

To understand this notice that, on the one hand, θ big implies firms underinvest in the CM because they get a good deal buying k in the DM. Higher ι counters this by effectively taxing the secondary market, leading to more primary investment and higher welfare. Then notice, on the other hand, θ small implies firms overinvest in the CM because they get a good deal selling k in the DM. Higher ι again counters this by taxing secondary trade, leading to less primary investment and higher welfare. Clearly, $\iota^* > 0$ is optimal for different reasons in the two cases – it mitigates underinvestment when θ is big and mitigates overinvestment when θ is small. In neither case do we get (k^*, q^*), however, since $\iota > 0$ precludes efficient reallocation by (34). Also, for $f(k) = k^\eta$ the Friedman rule is locally optimal for $\theta \in (\theta_L, \theta_H)$, but we cannot show it is globally optimal, since welfare may not be concave (although it was always concave in examples).

Of course the more interesting finding is not that $\iota = 0$ may be optimal, but that $\iota = 0$ is definitely not optimal for many parameters, i.e. for $\theta \notin (\theta_L, \theta_H)$. It is also worth highlighting that there is an asymmetry between monetary and fiscal policy. For fiscal policy, with $\iota = 0$, underinvestment implies $\tau^* < 0$ and overinvestment implies $\tau^* > 0$, which is not too surprising. However, for monetary policy, with $\tau = 0$, underinvestment and overinvestment both imply $\iota^* > 0$, which may be surprising. In any case, these results are novel, but accord well with intuition once the mechanics of investment, reallocation and liquidity are understood.

6 Competitive Search

Instead of random search and bargaining, here we study directed search and price posting, focusing mainly on monetary economies but breifly mentioning pure credit, too. The combination of directed search and posting is commonly called *competitive search equilibrium* (see the survey by Wright et al. 2018). This reflects the idea that agents posting the terms of trade compete to attract customers, where posting means commitment to these terms, which is very relevant in the presence of holdup problems like those discussed above. This seems interesting for secondary capital markets, as it is in labor, housing and other areas – there are different ways of conceptualizing markets with frictions, and each may capture more or less well important features in different applications.[19]

One way to formalize competitive search is to let sellers post the terms of trade, then buyers see what is posted and direct their search anywhere they like; another is to let buyers post and sellers search. These typically give the same outcome, but there are exceptions. One example is the model analyzed in this model, which will be discussed in more details later. Also see Delacroix and Shi (2017) for another examples.[20] In any case, the set of agents on one side posting the same terms, plus the set on the other side directing their search toward them, is called a submarket. Although agents can direct their search to any submarket, within a submarket there is still bilateral random meetings: $\alpha_s = \alpha(n)$ is again

[19]Commitment here means that sellers do not renegotiate when in contact with buyers; this is of course *not* the same as buyers committing to honor debt obligations, which would make credit viable. Hence, there is no inconsistency in our assumptions about commitment.

[20]There is a third standard way to describe competitive search that involves third parties, called market makers, setting up submarkets where they post terms to attract both buyers and sellers, the idea being that they can charge fees to get in, but competition drives the fees to 0. Usually the outcome is the same as having buyers or sellers post, but it raises issues here related to a point made by Faig and Huangfu (2007). They show market makers can do better than merely posting terms in a monetary economy with $\iota > 0$. Here is their arrangement: Market makers ask all buyers to hand over their cash at the entrance to their submarket. Then, if a buyer and seller meet, the latter gives the former what he wants for free. Then the market maker pays sellers from his gate receipts as they exit. This is better than buyers paying sellers directly, as that implies a dead weight loss associated with cash in the hands of potential buyers who do not meet sellers, which is costly if $\iota > 0$. This has market makers acting like the bankers in Berentsen et al. (2007), which is not what we want, because it changes the model beyond replacing random search and bargaining with directed search and posting. Hence, we rule out the scheme as in Rocheteau and Wright (2005) by assuming market makers cannot tell buyers from sellers. A related issue here is that market makers can potentially go beyond acting like banks and start acting like insurance companies: Agents visit a submarket before ε is realized. Then, after the realizations, they match bilaterally, and if one with ε_H meets one with ε_L the latter again gives the former q for free. This obviates the need for money completely, but only works if agents can commit to deliver q when asked, and that we rule out. Still, this should be studied further in future work – e.g., one idea is to model a market makers like the mechanism designer in Hu et al. (2009).

the probability a seller meets a buyer; $\alpha_b = \alpha(n)/n$ is the probability a buyer meets a seller; and again $n = n_b/n_s$ is market tightness, but now in the submarket, where $\alpha(n)$ satisfies the conditions in fn. 15. In general, note that agents may not direct their search to a submarket posting the best terms of trade, since they also take into account the probability of trade, determined by n.

To proceed, assume that buyers post, and given the parameterization in Section 5 the buyers are the firms with ε_H. They do so in the DM, after ε has been realized, of course, since only then does it make sense to say who is a buyer or a seller. When terms are posted in the DM, therefore, agents have already decided on z and k in the CM. Also, anticipating results, in equilibrium all submarkets are the same, since all buyers are the same and all sellers are the same, and hence $n = \gamma_H/\gamma_L$; but to find equilibrium we must first let n be anything, then equilibrate the system by setting $n = \gamma_H/\gamma_L$ (like Walrasian market clearing). For the terms of trade, suppose buyers post (p, q, n), which means this: when buyers meet sellers in this submarket, they trade q units of capital for a payment p, and the submarket has tightness n, although as usual it is not actually important to post n, since agents can always figure it out from p and q.

The buyer's DM problem is then

$$\begin{aligned} v_b = \max_{p,q,n} \frac{\alpha(n)}{n} A\left\{(1-\tau)\varepsilon_H\left[f(k_b+q) - f(k_b)\right] - p\right\} \\ \text{st } \alpha(n) A\left\{p - \varepsilon_L(1-\tau)\left[f(k_s) - f(k_s - q)\right]\right\} = v_s, \end{aligned} \tag{37}$$

where lower case v_j is the per period payoff. The constraint says buyers only get sellers to search for them if they match their market payoff v_s, which is taken as given by individuals, but endogenous in equilibrium (like Walrasian prices). There is another constraint, $z \geq p$, since buyers cannot pay more than they have, but without loss of generality we can set $z = p$, since they never bring more than they want to pay. Then the Lagrangian for (37) is

$$\begin{aligned}\mathcal{L} &= \frac{\alpha(n)}{n}A\left\{(1-\tau)\varepsilon_H\left[f(k_b+q)-f(k_b)\right]-z\right\} \\ &+\lambda\left\{\alpha(n)Az-\alpha(n)A\varepsilon_L(1-\tau)\left[f(k_s)-f(k_s-q)\right]-v_s\right\}.\end{aligned} \tag{38}$$

The FOC's are

$$0=\frac{\alpha(n)}{n}\varepsilon_H f'(k_b+q)-\lambda\alpha(n)\varepsilon_L f'(k_b-q) \tag{39}$$

$$0=\frac{\alpha(n)-n\alpha'(n)}{n^2}\left[\varepsilon_H f(k_b+q)-\varepsilon_H f(k_b)-\frac{z}{1-\tau}\right] \tag{40}$$

$$-\lambda\alpha'(n)\left[\frac{z}{1-\tau}-\varepsilon_L f(k_s)+\varepsilon_L f(k_s-q)\right]$$

$$v_s=\alpha(n)\left[\frac{z}{1-\tau}-\varepsilon_L f(k_s)+\varepsilon_L f(k_s-q)\right]. \tag{41}$$

Using (39) to eliminate λ from (40), we obtain

$$\begin{aligned}\frac{z}{1-\tau} &= \frac{e(n)\varepsilon_H f'(k_b+q)\varepsilon_L\left[f(k_s)-f(k_s-q)\right]}{e(n)\varepsilon_H f'(k_b+q)+\left[1-e(n)\right]\varepsilon_L f'(k_s-q)} \\ &+\frac{\left[1-e(n)\right]\varepsilon_L f'(k_s-q)\varepsilon_H\left[f(k_b+q)-f(k_b)\right]}{e(n)\varepsilon_H f'(k_b+q)+\left[1-e(n)\right]\varepsilon_L f'(k_s-q)}\end{aligned} \tag{42}$$

where $e(n)$ is the elasticity of the matching function. It is interesting (although common in related models) to note that (42) says the payment $p=z$ is the same as as the outcome of Nash bargaining when buyer's bargaining power is $\theta=e(n)$. Given this, n and q solve (41) and (42).

DM value function is Because $V(z,k,\varepsilon_H)=v_b+W(z,k,\varepsilon_H)$, by the envelope theorem

$$\begin{aligned}V_1(z,k,\varepsilon_H) &= \frac{\alpha(n)}{n}A\frac{\varepsilon_H f'(k+q)-\varepsilon_L f'(k-q)}{\varepsilon_L f'(k-q)}+A \\ V_2(z,k,\varepsilon_H) &= \frac{\alpha(n)}{n}A(1-\tau)\left[\varepsilon_H f'(k+q)-\varepsilon_H f'(k)\right] \\ &+A(1-\tau)\left[\varepsilon_H f'(k)+1-\delta\right].\end{aligned}$$

From the expression for $V(z,k,\varepsilon_L)$ we obtain

$$\begin{aligned} V_1(z,k,\varepsilon_L) &= A \\ V_2(z,k,\varepsilon_L) &= \alpha(n)\,\varepsilon_L A(1-\tau)\left[f'(k-q)-f'(k)\right] \\ &\quad + A(1-\tau)\left[\varepsilon_L f'(k)+1-\delta\right]. \end{aligned}$$

Combined with the FOC's from the CM problem, we arrive at

$$\iota = \frac{\alpha(n)}{n}\gamma_H \frac{\varepsilon_H f'(k+q)-\varepsilon_L f'(k-q)}{\varepsilon_L f'(k-q)}, \tag{43}$$

$$\frac{r+\delta}{1-\tau} = Cf'(k)+\xi\varepsilon_H f'(k+q)+\xi\varepsilon_L f'(k-q). \tag{44}$$

Monetary equilibrium with competitive search can be defined similarly to equilibrium with random search and bargaining. Assuming a quasi-linear technology, and restricting attention to steady state, it comes from to a pair (k,q) solving (43)-(44). This leads to the following result:

Proposition 8 *With our convenient parameterization and competitive search, monetary steady state exists iff* $\iota < \hat{\iota} \equiv \alpha_H\gamma_H(\varepsilon_H-\varepsilon_L)/\varepsilon_L$. *When it exists it is unique.*

As in Proposition 5, a monetary equilibrium exists only if ι is below a threshold, but now the threshold is bigger unless $\theta = 1$. Thus, competitive search allows monetary equilibrium to exist for more parameter values, because it is a more efficient trading arrangement. Moreover, in Proposition 5 we need parameter conditions for uniqueness (ι not too high or θ not too low), while there are no such conditions with competitive search (equilibrium is always unique).

The assumption that buyers post in the DM is crucial to the uniqueness result. It guarantees that a firm endogenizes the effect of his money holding on the terms of trade in DM. Hence, he trade off the cost of holding cash and the benefit of better terms of trade, which delivers uniqueness. If, instead, sellers post, they take the buyers' money holding as given and only post a payment equal to buyers' money holding. At the same time, firms anticipate the terms of trade in the DM and would only bring in enough cash to cover the payment. This gives rise to a coordination problem that leads to multiple equilibria.

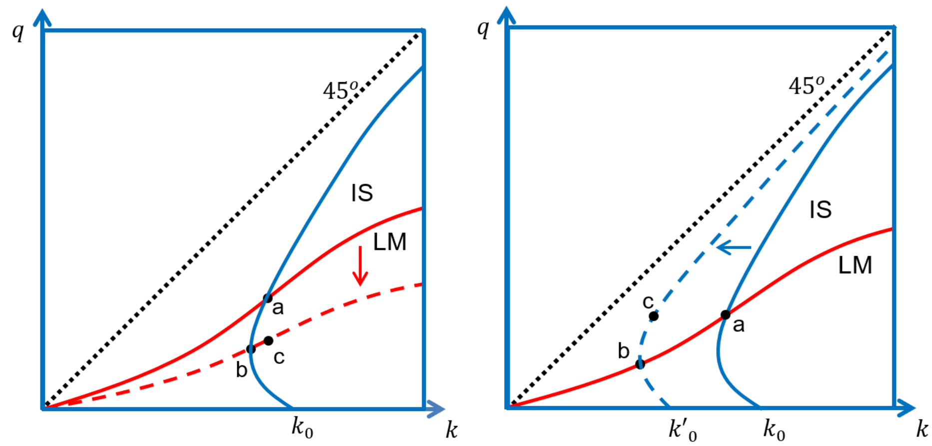


Figure 7: IS and LM with competitive search, monetary and fiscal policy

Figure 7 shows the IS and LM curves with competitive search. Notice the IS curve is nonmonotone in this model, but we still have uniqueness. In terms of policy, we can show (see the Online Appendix) that under reasonable conditions increases in ι or τ reduce k and q.[21] The next result is proved simply by comparing (43)-(44) to the planner's solution.

Proposition 9 *Consider our convenient parameterization with competitive search and $\tau = 0$. Then monetary equilibrium is efficient iff $\iota = 0$.*

Given this, next result is obvious:

Proposition 10 *Consider our convenient parameterization with competitive search and $\tau = 0$. Then pure-credit equilibrium is efficient.*

[21] Given the parameterization in fn. 18 and Condition F, we can also show that increases in B, γ_H or ε_H raise k and q, while increases in ε_L raise k but lower q.

To understand these results, first notice that (43) looks like what we get with Nash bargaining if the buyer has all the bargaining power, which is what it takes to get the efficient q at $\iota = 0$ (i.e., that is what it takes under Nash bargaining; Kalia delivers the efficient q at $\iota = 0$ for any $\theta > 0$ such that monetary equilibrium exists). Moreover, (44) looks like what we get with Nash bargaining if the buyer and seller both have all the bargaining power – which is not possible, of course, but we mean only that there are no holdup problems hindering inverstment in either money or capital, and that is what it takes to get efficiency. Hence, competitive search is clearly a better way to organize markets, attriibutable at least in part to commitment, but assuming this in a model does not make limited commitment problems go away in the real world. Our goal here is to sort out logically what happens under different market structures and policies.[22]

7 Numerical Analysis

Here we discuss additional features using numerical examples, mainly using bargaining (we can get similar results with posting). To begin, we claim the model is consistent with a few stylized facts deemed interesting in the literature: capital reallocation is procyclical; capital mismatch is countercyclical; the price of capital in the secondary market is procyclical; and spending on capital in this market, as a fraction of total investment, is procyclical. To analyze these facts, we appeal to the findings in Kehrig (2015) that say the dispersion in the productivity distribution is countercyclical, with firms at the lower end more effected by business cycles.[23] We take this as exogenous, and check if the model can generate

[22] Actual capital reallocation is probably somewhere in between, with some trade more accurately described by random search and bargaining, and other trade better described by direced search and posting. Combining them in one model, with the shares disciplined by data, is feasible (Bethune et al. 2018 do this for consumption goods markets), but beyond the scope of the current paper.

[23] As Kehrig (2015) says: "First, crosssectional productivity dispersion is countercyclical; the distribution of total factor productivity levels across establishments is about 12% more spreadout in a recession than in a boom. Second, the bottom quantiles of the productivity distribution

the stylized facts. Intuitively, the reason it could is that higher ε_H and ε_L lead to higher q, so there is more reallocation in good times, while as long as ε_L goes up more than ε_H mismatch is lower in good times.

Figure 4 presents an example showing how ε_L affects k, q, $P = p/q$ and p/k, as well as welfare W measured in the standard way.[24] It also shows three measures of dispersion: (i) the cross-sectional standard deviation of marginal product, MPK; (ii) cross-sectional standard deviation of $\log MPK$; (iii) cross-sectional standard deviation of MPK normalized by average MPK; and in what follows these are denoted ξ, $\hat{\xi}$, and $\tilde{\xi}$, respectively. As shown, good times are associated with less mismatch by all three measures. Still, good times have more reallocation measured by DM spending p, by the quantity of capital traded q, and by spending as a fraction of the stock p/k. Good times also have a higher unit price of capital, $P = p/q$. So although we do not consider this a serious calibration exercise, it shows that in principle the model can easily match the stylized facts.

are more cyclical than the top quantiles. In other words, the countercyclicality of productivity dispersion is mostly due to a higher share of relatively unproductive establishments during downturns." While we do not calibrate to match these numbers, we use the general idea.

[24]The example uses $u(x) = 4x^{0.5}$, $f(k) = k^{0.3}$, $\alpha = 0.5$, $\gamma_H = \gamma_L = 0.5$, $\beta = 0.98$, $\delta = 0.05$, $\tau = 0$, $\theta = 1$, $A = B = C = 1$, $\iota = 0.01$ and $\varepsilon_H = 2.3$, while ε_L varies along the horizontal axis.

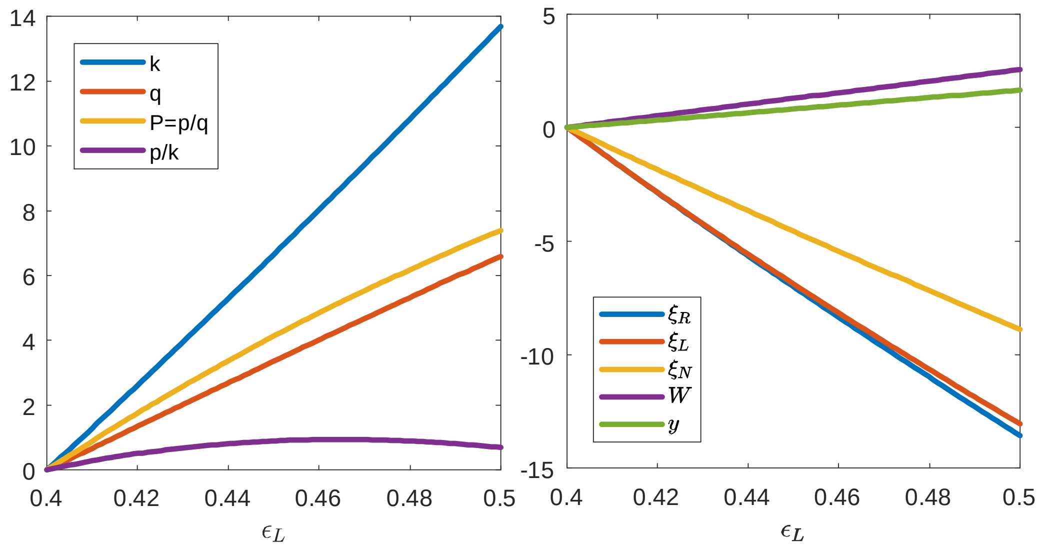


Figure 4: Volume, Price and MPK Dispersion.

It is standard (e.g., Eisfeldt and Rampini 2006 or Hsieh and Klenow 2009) to interpret misallocation as meaning marginal products are not equalized across firms. It is therefore tempting to think our statistics ξ, $\hat{\xi}$, and $\tilde{\xi}$ are positively associated with some notion of frictions and negatively associated with welfare – but that need not be true. Obviously $\iota > 0$ is a financial fiction, making liquidity constraints tighter. Figures 5 and 6 show, for different θ, how ι affects these measures, plus k, q, p/q, p/k and W.[25] With $\theta = 1$, notice ξ need not increase with ι, because higher ι reduces q, but it also increases k (with $\theta = 1$) and that homogenizes MPK (e.g., if all firms have very big k their marginal products are all close to 0). If the latter effect dominates, which it does at $\theta = 1$, then ξ decreases with ι. However, if θ is small then, as shown in Figure 6, both k and q decrease while ξ increases with ι. The measure $\hat{\xi}$ is even worse at capturing frictions, decreasing in ι in both examples. The measure $\tilde{\xi}$ performs better, increasing in ι in the examples, because it corrects for homogenization

[25] The example uses $u(x) = 4x^{0.5}$, $f(k) = k^{0.3}$, $\alpha = 0.5$, $\gamma_H = \gamma_L = 0.5$, $\beta = 0.98$, $\delta = 0.05$, $\tau = 0$, $A = B = C = 1$, $\varepsilon_L = 0.4$ and $\varepsilon_H = 3$, while ι varies along the horizontal axis, and θ values are indicated in the captions.

from higher k.

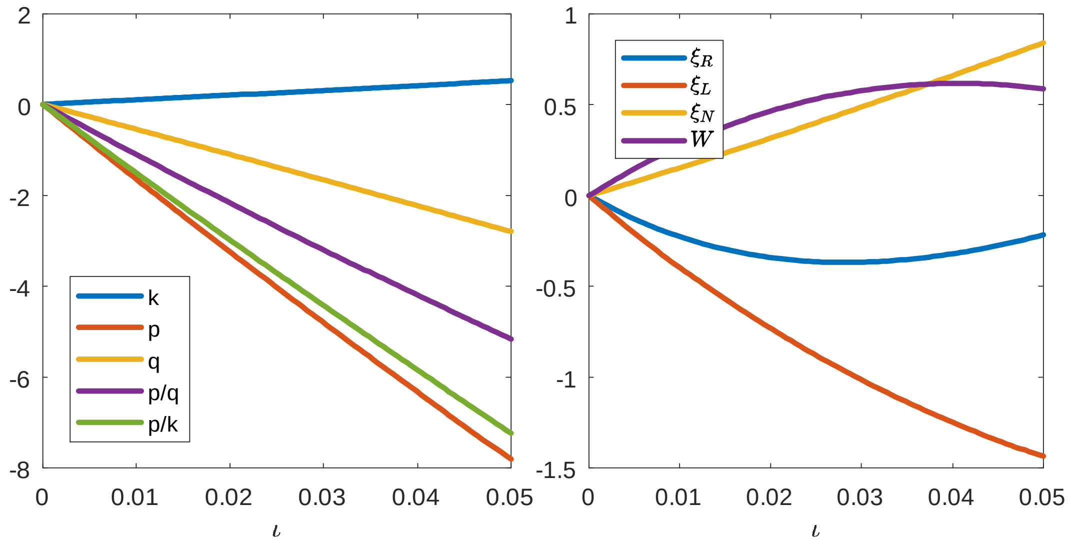


Figure 5: Misallocation, k, q,P, z/k and W vs ι with $\theta = 1$

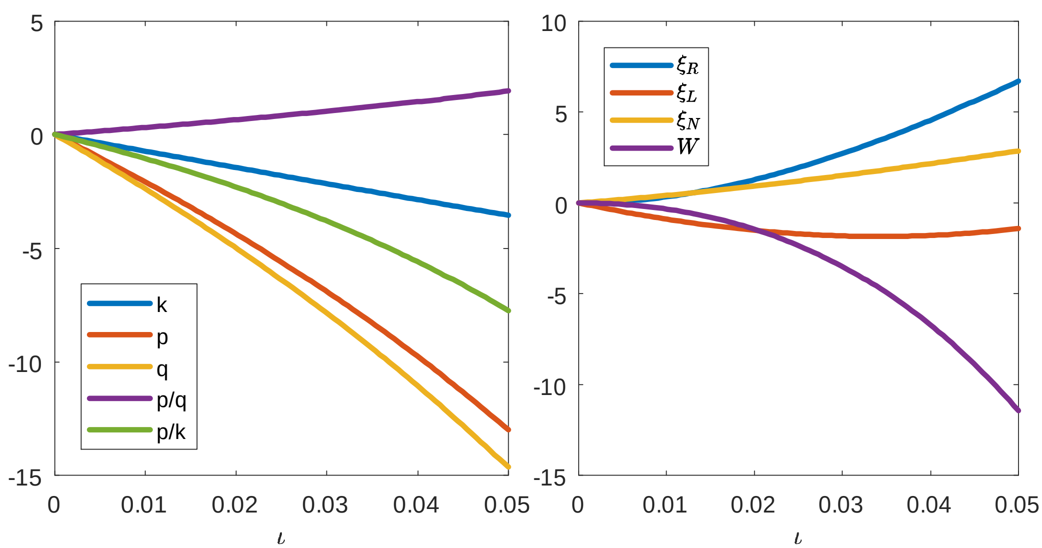


Figure 6: Misallocation, k, q, y and welfare vs ι with $\theta = 0.423$.

However, when it comes to welfare, none of these misallocation measures do well. In Figure 5, W increases for ι between 0.03 and 0.04 while ξ and $\tilde{\xi}$ are increasing. In Figure 6, W is globally decreasing in ι while $\hat{\xi}$ is not. One can also document similar results for the search frictions and other market imperfections. The general point is that in an economy with fundamental frictions, higher dispersion in MPK or $\log MPK$ does not necessarily correspond to lower efficiency or higher frictions, and while dispersion in MPK divided by average MPK may be a better measure of frictions, it is not necessarily a good measure of welfare. To be clear, we are not saying that more dispersion in MPK is always associated with lower frictions or higher welfare, only that this is possible.[26]

Note that in the bargaining model $\theta = 1$ implies W is increasing in ι at $\iota = 0$, so the optimal policy is not the Friedman rule. This is because there is under accumulation of capital when θ is high, and that potentially makes $\iota > 0$ beneficial by encouraging CM investment. Alternatively, we can subsidize investment by lowering τ below 0, its value in this example. Given $\tau = 0$, optimal monetary policy is achieved at $\iota = 4.1\%$, corresponding to an inflation rate of $\pi = 2\%$. While by no means a serious calibration, the example uses reasonable parameters, and delivers realistic nominal interest and inflation rate targets. Having said that, if instead of $\theta = 1$ we use $\theta = 0.423$, the value defined by (36), the optimal policy is $\iota = \tau = 0$. Hence optimal policy depends critically on the assumed market structure as well as parameter values.

8 Conclusion

This paper explored the determination of capital investment and reallocation in dynamic general equilibrium. The theory included frictional secondary markets with credit or monetary exchange, and different microstructures including random search and bargaining plus directed search and posting. For each specification we provided relatively strong results on existence, uniqueness, efficiency and policy. The framework is tractable: it can be reduced to two equations for capital

[26]This is somewhat related to Asker et al. (2014) who argue that dispersion in MPK could be purely an outcome of time-to-build and producitivity shocks. However, a frictionless secondary market would scuttle that explanation – another reason to consider frictional secondary markets.

and money – or, if one prefers, for investment and reallocation. Depending on parameters, decreasing the nominal interest rate can stimulate real investment and output, consistent with Keynesian macroeconomics, even if our approach to microfoundations is very different. In some versions of the model, inflation above the Friedman rule is optimal because, while it hinders the secondary market, it encourages investment in the primary market.

We also argued that common measures of mismatch related to productivity dispersion do not necessarily capture frictions. Further, we showed how to account for some stylized facts. All of these results help us better understand issues related to investment and reallocation, and to the effects of monetary and fiscal policy. In terms of future research, one could further pursue quantitative analysis. For this one should perhaps relax a few special assumptions – like i.i.d. shocks, or having only two realizations – that were made here to build simple examples illustrating the ideas. One can also add aggregate shocks. It might be interesting to additionally examine endogenous growth in this framework, perhaps allowing liquid assets other than currency to facilitate trade, and perhaps allowing financial intermediation. Additionally, it might be interesting to combine models with frictional capital and frictional labor markets. All of this is left for future work.

Appendix

Proof of Proposition 5: First, (34) defines q as a function of k, say $q = Q(k)$, as long as ι is not too big, where

$$\begin{aligned} Q(k) &= \frac{f'(k+q) f''(k-q) - f'(k-q) f''(k+q)}{f'(k+q) f''(k-q) + f'(k-q) f''(k+q)} \\ &\simeq \frac{\partial}{\partial q} \left[f'(k+q) f'(k-q) \right]. \end{aligned}$$

Although $f'(k+q) f'(k-q)$ is increasing in common examples like $f(k) = k^{\eta}$, it cannot be signed in general. Notice $Q(0) = 0$, $Q(k) > 0$ if $k > 0$, and $Q'(k) < 1$.

Similarly, (35) defines k as a function of q: $k = K(q)$. Let k_0 satisfy

$$\frac{r+\delta}{1-\tau} = (\gamma_H \varepsilon_H + \gamma_L \varepsilon_L) f'(k_0).$$

Then $K(0) = k_0$. In addition, let $\bar{k}$ satisfy

$$\begin{aligned} \frac{r+\delta}{1-\tau} &= \gamma_H \alpha_H \theta \varepsilon_H f'\left(2\bar{k}\right) + \gamma_H \varepsilon_H (1-\alpha_H) f'\left(\bar{k}\right) \\ &\quad + \frac{\gamma_L \alpha_L (1-\theta)}{\theta} \varepsilon_H f'\left(2\bar{k}\right) + \gamma_L \varepsilon_L f'\left(\bar{k}\right). \end{aligned}$$

Then $\bar{k} = K\left(\bar{k}\right)$. Any q solves $Q \circ K(q) = q$ is an equilibrium. Notice, $Q \circ K(0) = Q(k_0) > 0$ and $Q \circ K\left(\bar{k}\right) = Q\left(\bar{k}\right) < \bar{k}$. Then by continuity, there exists at least one equilibrium..

If $\theta = 1$, one can easily check the FOC's are sufficient. Therefore, (k, q) is a steady state equilibrium iff it satisfies

$$\iota = \alpha_H \gamma_H \left[\frac{\varepsilon_H f'(k+q)}{\varepsilon_L f'(k-q)} - 1 \right] \tag{45}$$

$$r + \delta = \gamma_H \alpha_H \varepsilon_H f'(k+q) + (1-\alpha_H) \gamma_H \varepsilon_H f'(k) + \gamma_L \varepsilon_L f'(k). \tag{46}$$

Uniqueness follows if $K'(q) Q' \circ K(q) < 0$ whenever $Q \circ K(q) = 0$. To check this, notice

$$\begin{aligned} K_2'(q) Q_1' \circ K_2(q) \simeq & -f'(k+q) f''(k-q) \gamma_H \alpha_H \varepsilon_H f''(k+q) \\ & - \gamma_H \alpha_H \varepsilon_H f''(k+q) f'(k-q) f''(k+q) \\ & - \left[f'(k+q) f''(k-q) + f'(k-q) f''(k+q) \right] \\ & \times \left[(1-\alpha_H) \gamma_H \varepsilon_H f''(k) + \gamma_L \varepsilon_L f'(k) \right] < 0. \end{aligned}$$

This proves uniqueness.

$$\begin{aligned}
\iota &= \alpha_H \gamma_H \theta \left[\varepsilon_H f'(k+q) - \varepsilon_L f'(k-q)\right]/D && (47)\\
\frac{r+\delta}{1-\tau} &= \gamma_H \varepsilon_H \left[\alpha \gamma_L f'(k+q)\,\Omega(k,q) + (1-\alpha\gamma_L) f'(k)\right] && (48)\\
&\quad + \gamma_L \varepsilon_L \left[\alpha \gamma_H f'(k-q)\,\Gamma(k,q) + (1-\alpha\gamma_H) f'(k)\right],
\end{aligned}$$

$$\begin{aligned}
\Omega(k,q) &= \left[(1-\theta)\,\varepsilon_H f'(k) + \theta \varepsilon_L f'(k-q)\right]/D\\
\Gamma(k,q) &= \left[(1-\theta)\,\varepsilon_H f'(k+q) + \theta \varepsilon_L f'(k)\right]/D
\end{aligned}$$

If $\theta > 0$ and $\iota = 0$, (34) and (35) reduces to

$$\begin{aligned}
0 &= \varepsilon_H f'(k+q) - \varepsilon_L f'(k-q)\\
\frac{r+\delta}{1-\tau} &= \gamma_H \left\{\alpha\gamma_L \left[(1-\theta)\,\varepsilon_H f'(k) + \theta\varepsilon_L f'(k-q)\right] + (1-\alpha\gamma_L)\,\varepsilon_H f'(k)\right\}\\
&\quad + \gamma_L \left\{\alpha\gamma_H \left[(1-\theta)\,\varepsilon_H f'(k+q) + \theta\varepsilon_L f'(k)\right] + (1-\alpha\gamma_H)\,\varepsilon_L f'(k)\right\}.
\end{aligned}$$

Partially differentiate the right hand side of the above equations with respect to q and k to obtain

$$\begin{aligned}
\hat{\mathbf{J}}_{11} &= \varepsilon_H f'(k+q) + \varepsilon_L f''(k-q)\\
\hat{\mathbf{J}}_{12} &= \varepsilon_H f''(k+q) - \varepsilon_L f''(k-q)\\
\hat{\mathbf{J}}_{21} &= \alpha\gamma_L\gamma_H \left\{(1-\theta)\left[\varepsilon_H f''(k) + f''(k+q)\right] - \theta\varepsilon_L \left[f''(k-q) - f''(k)\right]\right\}\\
\hat{\mathbf{J}}_{22} &= \alpha\gamma_L\gamma_H \left\{(1-\theta)\left[\varepsilon_H f''(k) + f''(k+q)\right] + \theta\varepsilon_L \left[f''(k-q) + f''(k)\right]\right\}\\
&\quad + \varepsilon_H\gamma_H (1-\alpha\gamma_L)\, f''(k) + \varepsilon_L\gamma_L (1-\alpha\gamma_H)\, f''(k).
\end{aligned}$$

Obviously, $\hat{\mathbf{J}}_{11} < 0$ and $\hat{\mathbf{J}}_{22} < 0$. Therefore, one can show that at the equilibrium k and q,

$$K_2'(q)\, Q_1' \circ K_2(q) - 1 = \frac{\hat{\mathbf{J}}_{12}}{\hat{\mathbf{J}}_{11}} \frac{\hat{\mathbf{J}}_{21}}{\hat{\mathbf{J}}_{22}} - 1 \simeq \hat{\mathbf{J}}_{12}\hat{\mathbf{J}}_{21} - \hat{\mathbf{J}}_{11}\hat{\mathbf{J}}_{22} < 0$$

which implies uniqueness follows for $\iota = 0$. Then by continuity, uniqueness holds for ι not too big. ■

Proof of Proposition 7: One can show that welfare is determined by

$$\begin{aligned} \boldsymbol{W} &= -(r+\delta)k + \gamma_H \varepsilon_H \alpha_H f(k+q) + \gamma_H \varepsilon_H [1-\alpha_H] f(k) \\ &\quad + \gamma_L \varepsilon_L \alpha_L f(k-q) + \gamma_L \varepsilon_L [1-\alpha_L] f(k), \end{aligned}$$

which is the total benefit of capital minus the cost. To show the optimal ι^* is positive for some θ, we only need check $(\partial \boldsymbol{W}/\partial \iota)|_{\iota=0} > 0$. Notice that

$$\frac{\partial \boldsymbol{W}}{\partial \iota}|_{\iota=0} = N_k \frac{\partial k}{\partial \iota}|_{\iota=0}$$

where N_k is the net marginal benefit of capital:

$$\begin{aligned} N_k &= -(r+\delta) + \gamma_H \varepsilon_H \alpha_H f'(k+q) + \gamma_H \varepsilon_H (1-\alpha_H) f'(k) \\ &\quad + \gamma_L \varepsilon_L \alpha_L f'(k-q) + \gamma_L \varepsilon_L (1-\alpha_L) f'(k). \end{aligned}$$

Notice $NB_k = 0$ if $\theta = \theta^*$. Now $\theta > \theta^*$ implies $k < k^*$ and $NB_k > 0$ and $\theta < \theta^*$ implies $k > k^*$. Therefore, $\theta < \theta^*$ implies $(\partial \boldsymbol{W}/\partial \iota)|_{\iota=0} > 0$ iff $(\partial k/\partial \iota)|_{\iota=0} < 0$, and $\theta > \theta^*$ implies $(\partial \boldsymbol{W}/\partial \iota)|_{\iota=0} > 0$ iff $(\partial k/\partial \iota)|_{\iota=0} > 0$.

Next, totally differentiate (34)-(35) and evaluate the result at the equilibrium (k, q) with $\iota = 0$. It turns out to be easier to rewrite (35) using $\iota = \gamma_H \alpha_H \Lambda(k, q)$ and obtain

$$\begin{aligned} r+\delta &= \alpha \gamma_H \gamma_L \theta \varepsilon_H f'(k+q) - \iota(1-\theta)\varepsilon_H [f'(k+q) - f'(k)] \\ &\quad + \alpha \gamma_L \gamma_H (1-\theta) \varepsilon_L f'(k-q) - \iota(1-\theta)\varepsilon_L [f'(k) - f'(k-q)] \\ &\quad + C f'(k). \end{aligned} \tag{49}$$

Then use (34) and (49) and the fact that $\varepsilon_H f'(k+q) = \varepsilon_L f'(k-q)$ to obtain, at $\iota = 0$,

$$\begin{bmatrix} \Upsilon_0 & \Upsilon_0 \\ \Upsilon_1 & \Upsilon_2 \end{bmatrix} \begin{bmatrix} \partial q \\ \partial k \end{bmatrix} = \begin{bmatrix} 1 \\ -(1-\theta)(\varepsilon_H - \varepsilon_L) f'(k) \end{bmatrix} \partial \iota$$

where

$$\begin{aligned} \Upsilon_0 &= \alpha \gamma_H \gamma_L \theta [\varepsilon_H f''(k+q) + \varepsilon_L f''(k-q)]/D \\ \Upsilon_1 &= \alpha \gamma_H \gamma_L [\theta \varepsilon_H f''(k+q) - (1-\theta)\varepsilon_L f''(k-q)], \\ \Upsilon_2 &= \alpha \gamma_H \gamma_L \theta \varepsilon_H f''(k+q) + \alpha \gamma_L \gamma_H (1-\theta) \varepsilon_L f''(k-q) \\ &\quad + C f''(k). \end{aligned}$$

The determinant of the Jacobian matrix is equal in sign to

$$\begin{aligned}
&\alpha\gamma_H\gamma_L\left[\varepsilon_H f''(k+q)+\varepsilon_L f''(k-q)\right]\left[\theta\varepsilon_H f''(k+q)+(1-\theta)\varepsilon_L f''(k-q)\right]\\
&-\alpha\gamma_H\gamma_L\Phi(k,q)\left[\theta\varepsilon_H f''(k+q)-(1-\theta)\varepsilon_L f''(k-q)\right]\\
&+\alpha\gamma_H\gamma_L C\left[\varepsilon_H f''(k+q)+\varepsilon_L f''(k-q)\right]f''(k).
\end{aligned}$$

One can check that the above expression is positive, so by Cramer's rule,

$$\begin{aligned}
\frac{\partial k}{\partial \iota}|_{\iota=0} \simeq\; & -\theta(1-\theta)\left[\varepsilon_H f''(k+q)+\varepsilon_L f''(k-q)\right](\varepsilon_H-\varepsilon_L) f'(k)\\
& -\left[\theta\varepsilon_H f''(k+q)-(1-\theta)\varepsilon_L f''(k-q)\right]\varepsilon_H f'(k+q).
\end{aligned}$$

The first term is positive and the second can be positive or negative depending on θ. If $\theta=1$, the first term is 0 while the last term is positive. By continuity, there exists $\theta_H>\theta^*$ such that $(\partial k/\partial\iota)|_{\iota=0}>0\ \forall\theta>\theta_H$ and hence $(\partial\mathbf{W}/\partial\iota)|_{\iota=0}>0$. Similarly, if $\theta=0$ then $(\partial k/\partial\iota)|_{\iota=0}<0$, and by continuity, $\theta<\theta_L\leq\theta^*$ implies $(\partial k/\partial\iota)|_{\iota=0}<0$ and $(\partial\mathbf{W}/\partial\iota)|_{\iota=0}>0$. In either case, $\iota>0$ yields higher welfare than $\iota=0$, but does not achieve the first best because q is not efficient.

Now suppose that $f(k)=k^\eta$. At $\iota=0$, one can show that $q=ck$ where

$$c=\frac{(\varepsilon_H/\varepsilon_L)^{\frac{1}{1-\eta}}-1}{(\varepsilon_H/\varepsilon_L)^{\frac{1}{1-\eta}}+1}=\frac{\varepsilon_H^{\frac{1}{1-\eta}}-\varepsilon_L^{\frac{1}{1-\eta}}}{\varepsilon_H^{\frac{1}{1-\eta}}+\varepsilon_L^{\frac{1}{1-\eta}}}.$$

From this we obtain

$$\begin{aligned}
\frac{\partial k}{\partial \iota}|_{\iota=0} \simeq\; & \theta(1-\theta)\left[\varepsilon_H(1+c)^{\eta-2}+\varepsilon_L(1-c)^{\eta-2}\right](\varepsilon_H-\varepsilon_L)\\
& +\left[\theta\varepsilon_H(1+c)^{\eta-2}-(1-\theta)\varepsilon_L(1-c)^{\eta-2}\right]\varepsilon_H(1+c)^{\eta-1}.
\end{aligned}$$

The RHS is a quadratic function of θ with roots

$$\begin{aligned}
\theta_1 &= \frac{1+\frac{\varepsilon_H(1+c)^{\eta-1}}{\varepsilon_H-\varepsilon_L}+\sqrt{\left[1+\frac{\varepsilon_H(1+c)^{\eta-1}}{\varepsilon_H-\varepsilon_L}\right]^2-4\frac{\varepsilon_H\varepsilon_L(1-c)^{\eta-2}(1+c)^{\eta-1}}{\left[\varepsilon_H(1+c)^{\eta-2}+\varepsilon_L(1-c)^{\eta-2}\right](\varepsilon_H-\varepsilon_L)}}}{2},\\
\theta_2 &= \frac{1+\frac{\varepsilon_H(1+c)^{\eta-1}}{\varepsilon_H-\varepsilon_L}-\sqrt{\left[1+\frac{\varepsilon_H(1+c)^{\eta-1}}{\varepsilon_H-\varepsilon_L}\right]^2-4\frac{\varepsilon_H\varepsilon_L(1-c)^{\eta-2}(1+c)^{\eta-1}}{\left[\varepsilon_H(1+c)^{\eta-2}+\varepsilon_L(1-c)^{\eta-2}\right](\varepsilon_H-\varepsilon_L)}}}{2}.
\end{aligned}$$

Notice that $\theta_1 > 1$ and $\theta_2 \in (0,1)$ if

$$\frac{\varepsilon_H \varepsilon_L (1-c)^{\eta-2} (1+c)^{\eta-1}}{\left[\varepsilon_H (1+c)^{\eta-2} + \varepsilon_L (1-c)^{\eta-2}\right] (\varepsilon_H - \varepsilon_L)} < \frac{\varepsilon_H (1+c)^{\eta-1}}{\varepsilon_H - \varepsilon_L}.$$

This condition is equivalent to

$$\frac{\varepsilon_L (1-c)^{\eta-2}}{\varepsilon_H (1+c)^{\eta-2} + \varepsilon_L (1-c)^{\eta-2}} < 1,$$

which always holds. Therefore, $(\partial k/\partial \iota)|_{\iota=0} > 0$ if $\theta \in (\theta_2, 1]$ and $(\partial k/\partial \iota)|_{\iota=0} < 0$ if $\theta \in [0, \theta_2)$.

We now show that $\theta_2 > \theta^*$. Notice

$$\begin{aligned}
\theta^* &= \frac{\varepsilon_H - \omega^{1-\eta}}{\varepsilon_H - \varepsilon_L}, \\
\theta_2 &= \frac{\varepsilon_H - \varepsilon_L + \omega^{1-\eta} - \sqrt{\left[\varepsilon_H - \varepsilon_L + \omega^{1-\eta}\right]^2 - 2\frac{\omega^{1-\eta}\varepsilon_H^{\frac{1}{1-\eta}}(\varepsilon_H - \varepsilon_L)}{\omega}}}{2(\varepsilon_H - \varepsilon_L)}.
\end{aligned}$$

where ω is defined in the statement of this Proposition. Therefore,

$$\theta_2 - \theta^* \simeq 3\omega^{1-\eta} - (\varepsilon_H + \varepsilon_L) - \sqrt{\omega^{2(1-\eta)} + (\varepsilon_H - \varepsilon_L)\left[2\omega^{1-\eta} + \varepsilon_H - \varepsilon_L - 2\frac{\omega^{1-\eta}\varepsilon_H^{\frac{1}{1-\eta}}}{\omega}\right]}.$$

This is positive if

$$3\omega^{1-\eta} - (\varepsilon_H + \varepsilon_L) > \sqrt{\omega^{2(1-\eta)} + (\varepsilon_H - \varepsilon_L)\left[2\omega^{1-\eta} + \varepsilon_H - \varepsilon_L - 2\frac{\omega^{1-\eta}\varepsilon_H^{\frac{1}{1-\eta}}}{\omega}\right]}.$$

After some algebra, one can show this is equivalent to

$$\left[2\omega^{1-\eta} - (\varepsilon_H + \varepsilon_L)\right]\left[4\omega^{1-\eta} - (\varepsilon_H + \varepsilon_L)\right] > -\omega^{1-\eta}\frac{\varepsilon_H^{\frac{1}{1-\eta}} - \varepsilon_L^{\frac{1}{1-\eta}}}{\omega}(\varepsilon_H - \varepsilon_L) + (\varepsilon_H - \varepsilon_L)^2. \tag{50}$$

By Hölder's inequality, $\omega^{1-\eta} \geq (\varepsilon_H + \varepsilon_L)/2$. This means that the LHS is positive.

For the RHS, notice

$$
\begin{aligned}
& -\omega^{1-\eta} \frac{\varepsilon_H^{\frac{1}{1-\eta}} - \varepsilon_L^{\frac{1}{1-\eta}}}{\omega} \left(\varepsilon_H - \varepsilon_L\right) + \left(\varepsilon_H - \varepsilon_L\right)^2 \\
\simeq \quad & -\omega^{1-\eta} \left(\varepsilon_H^{\frac{1}{1-\eta}} - \varepsilon_L^{\frac{1}{1-\eta}}\right) + \left(\varepsilon_H - \varepsilon_L\right) \omega \\
\leq \quad & -\frac{\left(\varepsilon_H + \varepsilon_L\right)}{2} \left(\varepsilon_H^{\frac{1}{1-\eta}} - \varepsilon_L^{\frac{1}{1-\eta}}\right) + \left(\varepsilon_H - \varepsilon_L\right) \left(\frac{\varepsilon_H^{\frac{1}{1-\eta}}}{2} + \frac{\varepsilon_L^{\frac{1}{1-\eta}}}{2}\right) \\
\simeq \quad & -\left(\varepsilon_H + \varepsilon_L\right) \left(\varepsilon_H^{\frac{1}{1-\eta}} - \varepsilon_L^{\frac{1}{1-\eta}}\right) + \left(\varepsilon_H - \varepsilon_L\right) \left(\varepsilon_H^{\frac{1}{1-\eta}} + \varepsilon_L^{\frac{1}{1-\eta}}\right) \\
= \quad & \frac{2}{\varepsilon_H^{\frac{1}{\eta-1}} \varepsilon_L^{\frac{1}{\eta-1}}} \left(\varepsilon_H^{-\frac{\eta}{1-\eta}} - \varepsilon_L^{-\frac{\eta}{1-\eta}}\right) < 0.
\end{aligned}
$$

Hence (50) holds. Then we have established that $\theta_2 > \theta^*$. ■

Proof of Proposition 8: First, q is a continuous function of k, $q = Q(k)$ where

$$
Q'(k) = -\frac{f''(k+q) f'(k-q) - f'(k+q) f''(k-q)}{f''(k+q) f'(k-q) + f'(k+q) f''(k-q)}
$$

can be positive or negative depends on the sign of $f''(k+q) f'(k-q) - f'(k+q) f''(k-q)$. In addition, if $k \to 0$, $Q(k) \to 0$ and if $k \to \infty$, $Q(k) \to \infty$ and $k - Q(k) \to \infty$. Equation (**??**) defines k as a continuous function of q: $k = K(q)$. This function may not be a monotone function because

$$
K'(q) \simeq -\left\{\frac{\alpha(n)}{n} \gamma_H \varepsilon_H f''(k+q) - \left[\frac{\alpha(n)}{n} \gamma_H + \iota\right] \varepsilon_L f''(k-q)\right\} \lesseqgtr 0.
$$

Notice that $K(0) = k_0$ where k_0 solves

$$
\frac{r+\delta}{1-\tau} = \left(\gamma_H \varepsilon_H + \gamma_L \varepsilon_L\right) f'(k_0).
$$

If $q \to \infty$, $K(q) \to \infty$ and $K_2(q) - q \to c < \infty$ where c solves

$$
\frac{r+\delta}{1-\tau} = \alpha(n) \gamma_L \varepsilon_L f'(c).
$$

Any k that satisfies $K \circ Q(k) - k = 0$ is an equilibrium. Notice $K \circ Q(0) = k_0 > 0$. In addition, if $k \to \infty$

$$
K \circ Q(k) - k = K \circ Q(k) - Q(k) + Q(k) - k \to c - \infty.
$$

This means that $K \circ Q(k) - k < 0$ for k sufficiently large. By the intermediate value theorem, an equilibrium exists. At the equilibrium k and $q = Q(k)$,

$$\begin{aligned}
& \frac{\partial}{\partial k}\left[K \circ Q(k) - k\right] \\
= \; & K' \circ Q(k) \, Q'(k) - 1 \\
\simeq \; & -\left\{\gamma_H \left[1 - \frac{\alpha(n)}{n}\right] \varepsilon_H + \gamma_L \left[1 - \alpha(n)\right] \varepsilon_L\right\} f''(k) \frac{\partial}{\partial k} f'(k+q) \, f'(k-q) \\
& -2 \frac{\alpha(n)}{n} \gamma_H \varepsilon_H f''(k+q) \frac{\partial}{\partial k} f'(k+q) \, f'(k-q) .
\end{aligned}$$

Notice that all terms are negative. Therefore, $K' \circ Q(k) \, Q'(k) - 1 < 0$. In this case, $K \circ Q(k) - k = 0$ for at most one k and uniqueness follows. ■

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